SCHOOL OF ENGINEERING
BANGKOK UNIVERSITY

Unit Four:

Magnetostatics

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4a. Magnetism

In the previous three units the focus was on static electrical charges and the force produced by their arrangement in space. How the force due to a distribution of a distribution of not moving charges affects another is expressed by the formation of an electric field. The electric field is assumed to be generated by the source charges and causes a force on the remote charge through the relation $\overline{F} = q\overline{E}$ where q is the amplitude of the remote charge and E is the field generated by the source. In that analysis we did not mention the effect of the movement of the charges on the force and this movement affects remote moving charges.

Moving charges

When a charge moves its location in space becomes time dependent. Hence we can define charge's velocity and acceleration as

$$\overline{v}(t) = \frac{d\overline{r}(t)}{dt} \quad (4.1a)$$

$$\overline{a}(t) = \frac{d^2\overline{r}(t)}{dt^2} \quad (4.1b)$$

In the magnetostatic limit, the charges are assumed to move with constant velocity, or the charges reach a steady state where their velocities become constant, $\overline{v}(t) = \overline{v}$. The acceleration then becomes zero, $\overline{a}(t) = 0$ as acceleration is the time derivative of velocity. This model is illustrated in figure 4.1.

In this scenario, finding an expression for the force between the two charges might not be as straightforward as before. However for the purpose of understanding we consider a much simpler situation where the two charges are moving with the exact same velocity along the x axis, $\overline{v}_1 = \overline{v}_2 = -v\hat{x}$. The displacement vector between the two charges is along the y axis as shown in figure 4.2. Here the sign is negative as the velocity is along the negative x direction. If we as observers move at the exact same velocity as the two charges then in our view both of them are static. The force between them then follows the electrostatic limit

$$\overline{F} = \frac{q_1 q_2}{4\pi\varepsilon_o r_{12}^2}\hat{r}_{12} \quad (4.2)$$

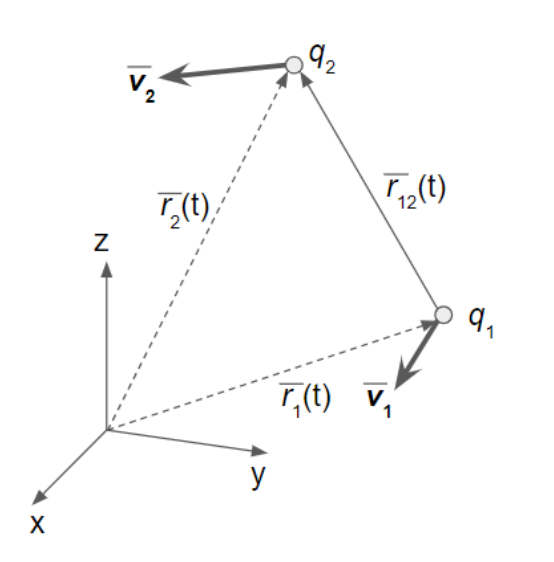


Figure 4.1. Two charges moving with constant velocities and displacement vectors that vary in time.

where, $\hat{r}_{12}$ is a unit vector that points from charge q_1 to q_2. In this special case it is along the y direction, $\hat{r}_{12} = \hat{y}$.

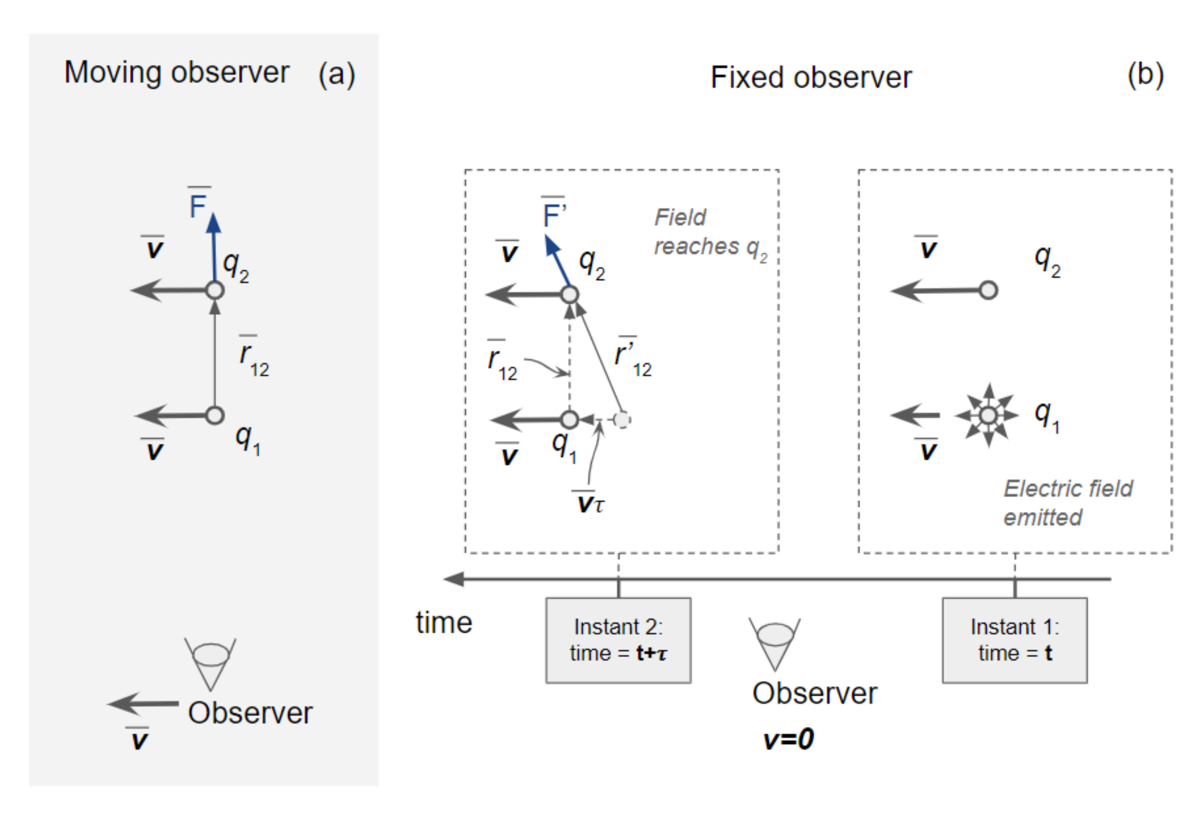


Figure 4.2. Special configuration of two moving charges. (a) When the observer is moving at the same velocity and (b) when the observer is fixed.

Notice here that the force has an amplitude $\frac{q_1 q_2}{4\pi\varepsilon_o r_{12}^2}$ and direction along the y axis as in figure 4.2a. The situation becomes different when the observer is fixed. In this case the observer monitors two instants. At the first instant, time = t in figure 4.2b, the observer sees that charge q_1 just emitted an electric field. At the second instant that is delayed by a time τ, $time = t + \tau$ in figure 4.2b, the field reaches q_2. During this delay τ, q_2 has been displaced by an amount, $\overline{\Delta r}$, that equals the multiplication of the charge velocity by the time delay, $\overline{v}\tau$. The electric field then that connects the old location of q_1 to the current location of q_2 experiences a displacement vectors $\overline{r'}_{12} = \overline{r}_{12} + \overline{\Delta r} = \overline{r}_{12} + \overline{v}t$ as shown in the geometry in figure 4.2b. As can be seen, the vector $\overline{r'}_{12}$ is not exactly along the y axis as it was the case in the electrostatic limit. In our special case we know that $\overline{v} =- v\widehat{x}$ and $\overline{r}_{12} = r_{12}\widehat{y}$. Hence, we can write the displacement vector as

$$\overline{r'}_{12} =- v\tau\,\widehat{x} + r_{12}\,\widehat{y} \tag{4.3}$$

The force in this situation is

$$\overline{F'} = \frac{q_1 q_2}{4\pi\varepsilon_o r'^2_{12}}\widehat{r'}_{12} \tag{4.4}$$

Comparing the two forces for the moving and the fixed observers on notice that for the case of the moving observer the force is inversely proportional to the square of r_{12} while in the case of fixed observer it is inversely proportional to the square of r'_{12}. From the geometry in figure 4.2b one can tell that $r'_{12} > r_{12}$ and hence $F' < F$. What could possibly make the force to change in the case of moving charges? To answer this question, one needs to find an expression for the amplitude of the displacement vector r'_{12} in terms of r_{12}. From equation 4.3, one can point out that

$$r'^2_{12} = v^2\tau^2 + r^2_{12} \quad (4.5)$$

The delay between the two instants can be estimated by dividing the distance the electric field covers by its speed. The distance in this situation is r'_{12}. Here, it is safe to assume that the electromagnetic fields travel at the speed of light c. Actually, later we will learn that light is commonly represented as an electromagnetic wave. The delay is then $\tau = r'_{12}/c$. Using this relation in 4.5 we obtain

$$r'^2_{12} = v^2 r'^2_{12}/c^2 + r^2_{12} \quad (4.6)$$

Rearranging equation 4.6,

$$r'^2_{12}\left(1 - v^2/c^2\right) = r^2_{12} \rightarrow \quad (4.7a)$$

$$r'_{12} = r_{12}/\sqrt{1 - v^2/c^2} \quad (4.7b)$$

$$r'_{12} = \gamma r_{12} \quad (4.7c)$$

, where $\gamma = 1/\sqrt{1 - v^2/c^2}$. This is known as the **Lorentz factor**, a quantity that expresses how much the measurements of time, length, and other physical properties change for an object that moves with constant speed. It is widely used in Einstein's special relativity, a rather more proper approach to handle this problem. Using the relation derived above in equation 4.4, the force becomes

$$\overline{F'} = \frac{q_1 q_2}{4\pi\varepsilon_o \gamma^2 r^2_{12}} \widehat{r'}_{12} \quad (4.8)$$

Expanding lorentz factor,

$$\overline{F'} = \frac{q_1 q_2}{4\pi\varepsilon_o r^2_{12}} \left(1 - \frac{v^2}{c^2}\right) \widehat{r'}_{12} \quad (4.9)$$

That gives us two force components

$$\overline{F'} = \frac{q_1 q_2}{4\pi\varepsilon_o r_{12}^2}\widehat{r'}_{12} - \frac{\mu_o q_1 q_2}{4\pi r_{12}^2} v^2 \widehat{r'}_{12} \quad (4.9)$$

Notice that in the second term in equation 4.9, we use the relationship between the speed of light and both permittivity and **permeability** in vacuum, $c = 1/\sqrt{\varepsilon_o \mu_o}$. Here, Permeability or μ is the measure of material magnetization under the application of a **magnetic field**. As in the case of electric permittivity, the reference value is the vacuum permeability $\mu_o \approx 4\pi \times 10^{-7}$ N/A^2 or Newton per Ampere square. We will get back to the magnetic field and current later in this unit. If we assume the charge's speed is much slower than the speed of light and hence the vector $\widehat{r'}_{12}$ is approximately pointing at the same direction as $\widehat{r}_{12}$ or $\widehat{r'}_{12} \approx \widehat{y}$. The force is now simplified to

$$\overline{F} \approx \frac{q_1 q_2}{4\pi\varepsilon_o r_{12}^2(t)}\widehat{y} - \frac{\mu_o q_1 q_2 v^2}{4\pi r_{12}^2(t)}\widehat{y} \quad (4.10)$$

The first term in equation 4.10 is the electrostatic force between the two charges. The second term represents a force that depends on the movement of the charges as this term vanishes when the charge stops, $v = 0$. This term is known as the magnetic force,

$$\overline{F}_B = -\frac{\mu_o q_1 q_2 v^2}{4\pi r_{12}^2(t)}\widehat{y}. \quad (4.11)$$

This is exactly the term that causes the force to change from the electrostatic case. As the charges movement is represented by vector quantitites. It will be more appropriate to express the magnetic force in terms of the velocity. If we pay close attention to the term $- v^2 \widehat{y}$ we can write it as the cross product of three vectors: $\overline{v} \times (\overline{v} \times \widehat{y})$. Notice in this special example $\overline{v} = - v\,\widehat{x}$ and hence,

$$\overline{v} \times (\overline{v} \times \widehat{y}) = - v\,\widehat{x} \times (- v\,\widehat{x} \times \widehat{y}) = v\,\widehat{x} \times (\widehat{z}\, v) = v^2\,\widehat{x} \times \widehat{z} = - v^2\,\widehat{y}.$$

We can now express the magnetic force as

$$\overline{F}_B = \frac{\mu_o q_1 q_2}{4\pi r_{12}^2(t)}\overline{v} \times \left(\widehat{r}_{12} \times \overline{v}\right) \quad (4.12)$$

Although the expression in 4.12 is the proper one for the magnetic force between two charges that move at the same velocities, we need to keep in mind that our approach to reach there might not be the most accurate one. It is however a simplified way that gives us an appreciation of the rise of the magnetic force due to the motion of the charges. We will however accept equation 4.12 as a general form of the magnetic force.

Magnetic force and magnetic field

In the example above we considered two charges moving at exactly the same velocity. The force between the two charges is written as a summation of two forces $\overline{F} = \overline{F}_E + \overline{F}_B$, where $\overline{F}_E$ refers to the electrostatic force. In the electrostatic force limit we assumed that one charge, say q_1, generates a field, $\overline{E}_1 = \frac{q_1}{4\pi\varepsilon_o r_{12}^2(t)}$, where the electric force $\overline{F}_E = q_2\overline{E}_1$ is exerted on q_2. For the magnetic field, we could as well state a similar statement. The moving charge q_1 generates a magnetic field

$$\overline{B}_1 = \frac{\mu_o q_1}{4\pi r_{12}^2(t)}\left(\overline{v} \times \hat{r}_{12}\right) \tag{4.13}$$

The magnetic field has units of **Tesla**.The field exerts a force on another moving charge $\overline{F}_B = q_2\overline{v}_2 \times \overline{B}_1$. In this case, we can write the total **electromagnetic** force exerted on a charge q_2 that moves with velocity $\overline{v}_2$ due the presence of an electric field,$\overline{E}_1$ and magnetic field $\overline{B}_1$ as

$$\overline{F} = q_2\overline{E}_1 + q_2\overline{v}_2 \times \overline{B}_1 \tag{4.14}$$

This is known as the **Lorentz force** or **electromagnetic force**. In general we could say that the force exerted on a charge q that moves with a constant velocity $\overline{v}$ in the presence of an electric field $\overline{E}$ and a magnetic field $\overline{B}$ is

$$\overline{F} = q\left(\overline{E} + \overline{v} \times \overline{B}\right) \tag{4.15}$$

4b. Biot-Savart law

Flow of charges

When considering moving charges, it can be more practical to consider a flow of charges moving on a linear trajectory rather than one charge. For instance let's assume a large number of charges moving in a conductive wire with constant velocity $\overline{v} = v\,\hat{x}$ and a linear charge density ρ_l C/m as shown in figure 4.3.

Consider an infinitesimal element dl along a wire. The total charge inside this segment is $q = \rho_l dl$. If we consider this element as a point charge, then we know from equation 4.13 that the observed magnetic field at point P is

$$d\overline{B} = \frac{\mu_o \rho_l dl}{4\pi r^2}\left(\overline{v} \times \hat{r}\right) \qquad (4.16)$$

Notice that in equation 4.16, we write $d\overline{B}$ instead of $\overline{B}$. That is due to the fact that this is the field produced by the infinitesimal charge component inside *dl*.

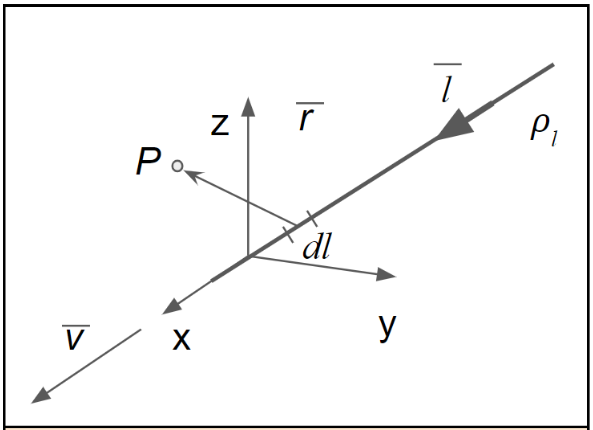


Figure 4.3. Magnetic field produced by a flow of charges along the x axis.

In the example presented, the charges are moving along the x axis. The displacement vector is $\overline{r} = \left(x - x_p, - y_p, - z_p\right)$ and its amplitude is $r = \sqrt{\left(x - x_p\right)^2 + y_p^2 + z_p^2}$. The unit vector $\hat{r} = \overline{r}/r$. Using these relations, the cross product $\overline{v} \times \hat{r}$ becomes

$$v\hat{x} \times \left(\frac{x-x_p}{r}\hat{x} + \frac{-y_p}{r}\hat{y} + \frac{-z_p}{r}\hat{z}\right) = v\left(0 - \frac{y_p}{r}\hat{z} + \frac{z_p}{r}\hat{y}\right) \qquad (4.17)$$

In deriving equation 4.15 the cross product relation (right hand rule) was used where $\hat{x} \times \hat{x} = 0$, $\hat{x} \times \hat{y} = \hat{z}$ and $\hat{x} \times \hat{z} = -\hat{y}$. The magnetic field has then two components one is in z direction and the other is in the y direction. The infinitesimal element is along the x-axis, $dl = dx$, hence the vector equation in 4.16 can be expanded as

$$dB_y = \frac{\mu_o v \rho_l}{4\pi r^2}\left(\frac{z_p}{r}\right)dx \qquad (4.18a)$$

$$dB_z = -\frac{\mu_o v \rho_l}{4\pi r^2}\left(\frac{z_p}{r}\right)dx \qquad (4.18b)$$

The magnetic field due to all the charges moving in the wire is

$$B_y = \frac{\mu_o v \rho_l}{4\pi}\int_{x=-\infty}^{\infty}\left(\frac{z_p}{r^3}\right)dx \qquad (4.19a)$$

$$B_z = -\frac{\mu_o v \rho_l}{4\pi}\int_{x=-\infty}^{\infty}\left(\frac{y_p}{r^3}\right)dx \qquad (4.19b)$$

Following the same approach as in deriving equation 1.17 in unit one, we obtain the following expression for the total magnetic field at point *P*.

$$B_y = -\frac{\mu_o v \rho_l z_p}{2\pi\left(z_p^2 + y_p^2\right)} \qquad (4.20a)$$

$$B_z = \frac{\mu_o v \rho_l y_p}{2\pi\left(z_p^2 + y_p^2\right)} \quad (4.20b)$$

The magnetic field as a vector is

$$\overline{B} = \frac{\mu_o v \rho_l}{2\pi R^2}\overline{u} \quad (4.20c)$$

where $\overline{u} = \left(0, -z_p, y_p\right)$. This vector has an amplitude $|\overline{u}| = \sqrt{z_p^2 + y_p^2}$. This is the same amplitude as the cylindrical displacement vector $\overline{R} = \left(0, y_p, z_p\right)$. The vector $\overline{u}$ is however is normal to $\overline{R}$:

$$\overline{R} \cdot \overline{u} = -y_p z_p + z_p y_p = 0.$$

As in fire 4.4, the magnetic field takes a circular path that is always normal to $\overline{R}$. In cylindrical coordinates, this vector is along the θ direction, $\hat{\theta} = \frac{\overline{u}}{R} \rightarrow \overline{u} = R\hat{\theta}$.

$$\overline{B} = \frac{\mu_o v \rho_l}{2\pi R^2} R\,\hat{\theta} = \frac{\mu_o v \rho_l}{2\pi R}\,\hat{\theta} \quad (4.21)$$

In equation 4.18 the quantity $v\rho_l$ has units of $(m/s).C/m$ or C/s, which is the total charge that flows in the wire per unit time. This quantity is commonly referred to as the **electric current**, $I = v\rho_l$ and the unit C/s is typically referred to as **Amperes**. Using this definition of electrical current, the magnetic field produced by charges flowing in a linear wire placed along the x-axis with a constant current, *I*, (i.e. constant speed v and a uniform linear charge density ρ_l) is

$$\overline{B} = \frac{\mu_o I}{2\pi R}\,\hat{\theta} \quad (4.22)$$

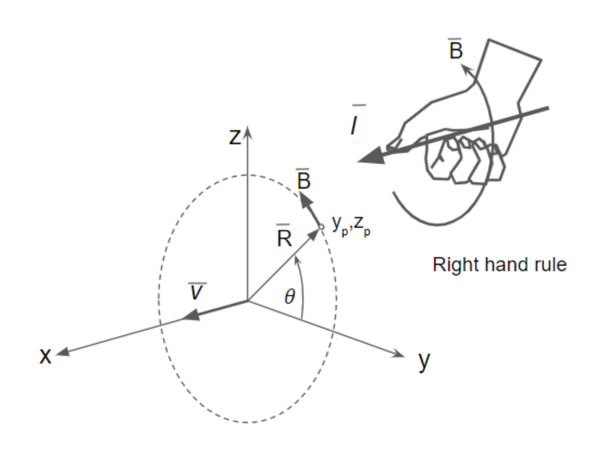


Figure 4.4. Magnetic field produced by constant current that flows along the x-axis.

The produced magnetic field is pointing in a circular direction around the wire and its amplitude is inversely proportional to the distance from the wire.

Wire of arbitrary shape

In the previous analysis we considered a special case where the constant electrical current is flowing in a straight wire that is aligned along the x-axis. However, we could still start from equation 4.16 to find the magnetic field caused by arbitrary shape wire as in figure 4.5.

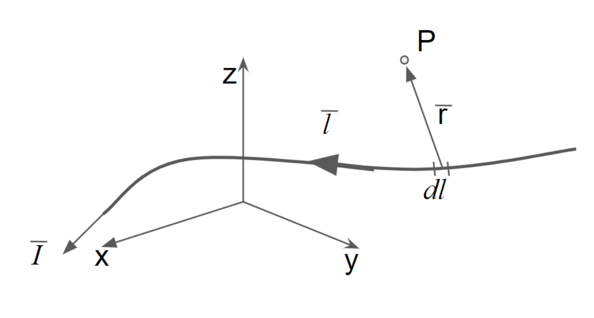


Figure 4.5. Constant current flowing through an arbitrary shape wire.

The current flows in a path $\bar{l}$ along the wire. Hence, the velocity of the elementary charge element is as well along the same direction, $\bar{v} = v\,\hat{l}$. The current can be written then as $\bar{I} = \rho_l \bar{v} = \rho_l v \hat{l}$. Notice that we wrote the current as a vector quantity. This is due to the fact that it flows along the path $\bar{l}$ of the moving charges. So we could write the current as $\bar{I} = I\,\hat{l}$.

The magnetic field due to the elementary charge element is then

$$d\bar{B} = \frac{\mu_o I dl}{4\pi r^2}\left(\hat{l} \times \hat{r}\right) \quad (4.23)$$

The magnetic field at point P is calculated by performing line integrating over the path $\bar{l}$

$$\bar{B} = \oint_l \frac{\mu_o I}{4\pi r^2}\left(\hat{l} \times \hat{r}\right) dl \quad (4.24)$$

Equation 4.23 is commonly known as **Biot-Savart law**. It describes the magnetic field generated by a constant electric current.

Magnetic field from a wire loop

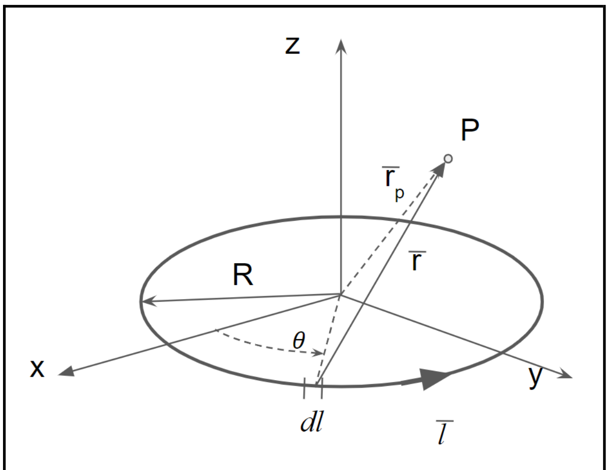


Figure 4.6. Current flowing in a circular loop.

In the previous section we considered a case where the line flows in a linear wire and we found that the generated magnetic field is pointing in a circular direction, $\hat{\theta}$. Now, let us consider the opposite scenario where the current is flowing in a circular loop of radius R as shown in figure 4.6. In order to calculate the magnetic field at point P, we apply Biot-Savart law in equation 4.24, where the integration is now over the circular path. In other words, $dl = Rd\theta$ and the current is $\bar{I} = I\,\hat{\theta}$. As a reminder when we use $\hat{}$ on top of a symbol it means a unit vector along that direction. Also the circular path is defined by

$x = Rcos\theta$, $y = Rsin\theta$ and $x^2 + y^2 = R^2$. Hence, the magnetic field is

$$\bar{B} = \int_{\theta=0}^{2\pi} \frac{\mu_o I}{4\pi r^2}\left(\hat{\theta} \times \hat{r}\right) R d\theta \quad (4.25)$$

Where $r = \sqrt{(x - x_p)^2 + (y - y_p)^2 + z_p^2}$. Expanding the square terms we can write $r = \sqrt{R^2 + r_p^2 - 2Rx_p cos\theta - 2Ry_p sin\theta}$, where $r = \sqrt{x_p^2 + y_p^2 + z_p^2}$. The unit vector $\hat{r}$ is

$$\hat{r} = \left(\frac{x-x_p}{r}, \frac{y-y_p}{r}, \frac{-z_p}{r}\right) \quad (4.26)$$

The circular unit vector is $\hat{\theta} = (-\,sin\theta\,, cos\theta\,, 0)$. Following the cross product introduction in unit three, the cross product $\hat{\theta} \times \hat{r}$ is

$$(-\,sin\theta\,, cos\theta\,, 0) \times \left(\frac{x-x_p}{r}, \frac{y-y_p}{r}, \frac{-z_p}{r}\right) = \left(\frac{-z_p cos\theta}{r}, \frac{-z_p sin\theta}{r}, \frac{(x-x_p)cos\theta-(y-y_p)sin\theta}{r}\right) \quad (4.27)$$

Replacing x and y in terms of R and θ, the cross product becomes

$$\hat{\theta} \times \hat{r} = \left(\frac{-z_p cos\theta}{r}, \frac{-z_p sin\theta}{r}, \frac{R-x_p cos\theta-y_p sin\theta}{r}\right) \quad (4.28)$$

The magnetic field is

$$B_x = \frac{\mu_o I}{4\pi}\int_{\theta=0}^{2\pi} \frac{-z_p cos\theta}{\left(R^2+r_p^2-2Rx_p cos\theta-2Ry_p sin\theta\right)^{3/2}} Rd\theta \quad (4.29a)$$

$$B_y = \frac{\mu_o I}{4\pi}\int_{\theta=0}^{2\pi} \frac{-z_p sin\theta}{\left(R^2+r_p^2-2Rx_p cos\theta-2Ry_p sin\theta\right)^{3/2}} Rd\theta \quad (4.29b)$$

$$B_z = \frac{\mu_o I}{4\pi}\int_{\theta=0}^{2\pi} \frac{R-x_p cos\theta-y_p sin\theta}{\left(R^2+r_p^2-2Rx_p cos\theta-2Ry_p sin\theta\right)^{3/2}} Rd\theta \quad (4.29c)$$

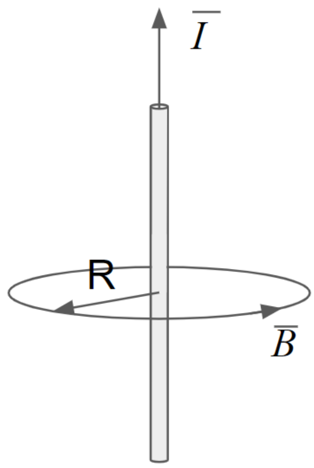
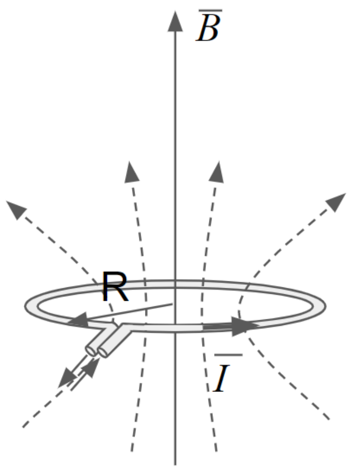
Performing the integrations in 4.29 can be challenging. However, we could consider a specific case where we observe the magnetic field at the axis of the loop, or the z-axis in our case. Hence, both x_p and y_p vanish. In this case the amplitude of the displacement vector becomes independent of θ, or $r = \sqrt{R^2 + z_p^2}$. The integrations in 4.29 become

$$B_x = \frac{\mu_o I}{4\pi}\frac{-z_p}{\left(R^2+z_p^2\right)^{3/2}} R \int_{\theta=0}^{2\pi} cos\theta d\theta = 0 \quad (4.30a)$$

$$B_x = \frac{\mu_o I}{4\pi}\frac{-z_p}{\left(R^2+z_p^2\right)^{3/2}} R \int_{\theta=0}^{2\pi} cos\theta d\theta = 0 \quad (4.30b)$$

$$B_z = \frac{\mu_o I}{4\pi}\frac{R^2}{\left(R^2+z_p^2\right)^{3/2}} \int_{\theta=0}^{2\pi} d\theta = \frac{\mu_o I R^2}{2\left(R^2+z_p^2\right)^{3/2}} \quad (4.30c)$$

The magnetic field at the axis of the loop is along the z-axis. Right at the central point of the loop, $z_p = 0$, the magnetic field has a maximum amplitude of $\frac{\mu_o I}{2R}$ and it reduces when moving away along the z axis. In table 4.1., we summarize the magnetic field produced by a linear wire and a loop.

Table 4.1. Summary of magnetic field in two different geometries, linear wire and loop.		
	Linear wire	Wire loop
Geometry		
Magnetic field amplitude	$\mu_o I/2\pi R$	At axis: $\frac{\mu_o I R^2}{2\left(R^2+z_p^2\right)^{3/2}}$ At center: $\frac{\mu_o I}{2R}$
Direction	Circular along $\hat{\theta}$	At axis: along $\hat{z}$

4.c Force on currents

Force on a moving charge

Electrical current typically flows in an electrical conductor wire that has a cylindrical shape with a specific radius. In many cases we can assume that the wire is very thin (the cross section is much smaller than the length) so that we can ignore the cylindrical cross section and only consider the value of the constant current that flows through a linear path that is formed by the wire.

If a charge q is moving with a velocity $\bar{v}$ near the wire as shown in figure 4.7, then accordion to lorentz force, it experiences a magnetic force that is equal to $\bar{F}_B = q\,\bar{v} \times \bar{B}$. If the conductor wire is considered to be placed along the z-axis such that the current I is flowing along the positive z-direction, then the magnetic force is

$$\bar{F}_B = q\,\bar{v} \times \left(\mu_o I/2\pi R\,\hat{\theta}\right) \quad (4.31)$$

If the charge was moving along the z-axis in the same direction as the current, $\bar{v} = v\,\hat{z}$, then the cross product term

$$\bar{v} \times \hat{\theta} = v\,(0, 0, z) \times (-\,sin\theta\,, cos\theta\,, 0)$$
$$= (-\,cos\theta\,, -\,sin\theta\,, 0) = -\,\hat{R} \quad (4.32)$$

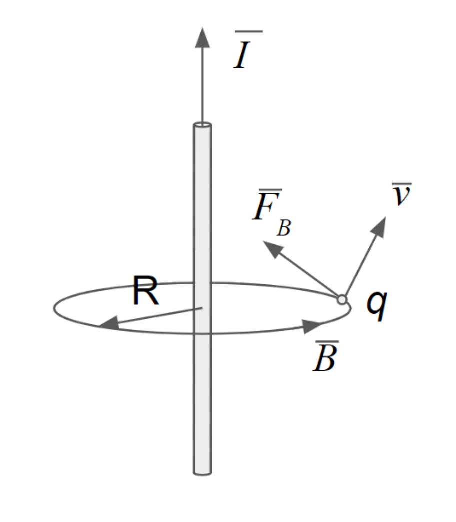


Figure 4.7. Magnetic force on a charge moving near a wire with constant current

The force vector is simplified to

$$\bar{F}_B = -\,q\,v\left(\mu_o I/2\pi R\right)\hat{R} \quad (4.33)$$

When the charge is moving in the same direction as the current (given that both the charges forming the current and q have the same sign) the resultant force attracts the charge q and pulls it towards the wire. The opposite however happens when the charge moves in the opposite direction. In that case, a repelling force is produced pushing the charge away from the wire as illustrated in figure 4.8.

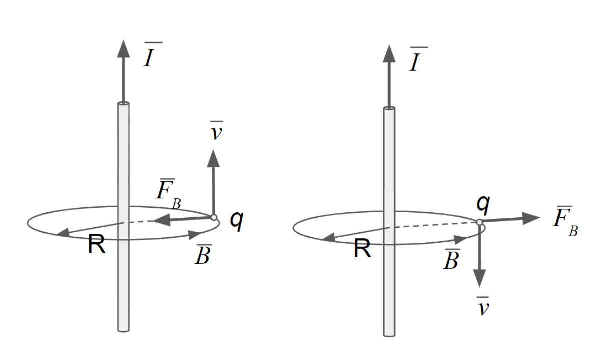


Figure 4.8. Attraction and repelling force between a current in a wire and a moving charge.

Constant currents in two parallel wires

The demonstration in figure 4.8 shows the effect of the force produced by a constant current on a charge moving with a constant velocity. If we replace the moving charge with a current flowing in a wire that is placed parallel to the original wire as shown in figure 4.9, then we could still use the relation in equation 4.30. However, in this case we would have to replace the charge q by an element charge $\rho_{l,2}dl$, where $\rho_{l.2}$ is the linear charge density of the second conductor wire.

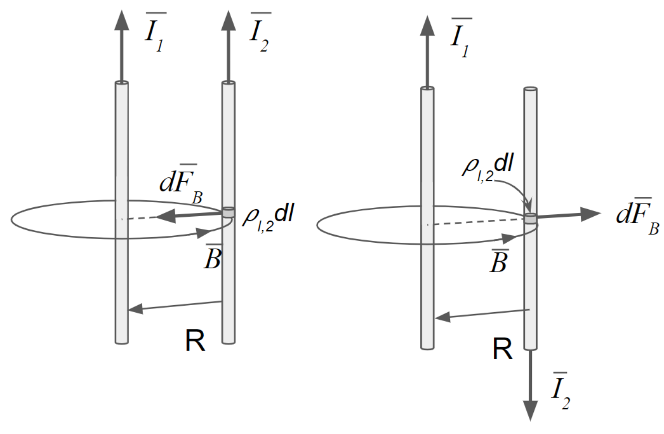


Figure 4.9. The magnetic force between two constant currents flowing in two parallel conductive wires.

$$d\overline{F}_B = -\rho_{l,2} dl\, v\left(\mu_o I_1/2\pi R\right)\widehat{R} \quad (4.34)$$

Here we denote the current in the original wire as I_1 in order to distinguish it from the second wire. Also, we use $d\overline{F}_B$ to indicate that the force is on the elementary charge across the second wire. Defining the current in the second wire as $I_2 = \rho_{l,2}v$, then the force becomes

$$d\overline{F}_B = -dl\, I_2\left(\mu_o I_1/2\pi R\right)\widehat{R} \quad (4.35)$$

In the case of long wires, it is more appropriate to represent the force per unit length or

$$\frac{d\overline{F}_B}{dl} = -\frac{\mu_o I_1 I_2}{2\pi R}\widehat{R} \quad (4.36)$$

Note that the same amplitude of the magnetic force is exerted on the original wire due to the current, I_2, flowing in the second wire. The direction however will be the opposite of the force exerted on the second wire. This can easily proven by moving the origin of the coordinates to the second wire and consider the magnetic field now is to be generated by I_2 and the elementary charge is on the first wire. Hence, we can state the following:

When two constant currents flow in two parallel conductive wires that are separated by a distance R, each wire experiences a magnetic force of amplitude

$$\frac{dF_B}{dl} = \mu_o I_1 I_2/2\pi R$$

The two wires are attracted to each other if both currents flow in the same direction, otherwise the wires repel.

If one of the wires, say the second one, has a finite length L, then the total force on that wire segment can be calculated by integrating the force over that segment

$$F_B = \int_{l=0}^{L} \frac{\mu_o I_1 I_2}{2\pi R}\, dl = \frac{\mu_o I_1 I_2 L}{2\pi R} \quad (4.37)$$