Part 1: Moving charges and radiation

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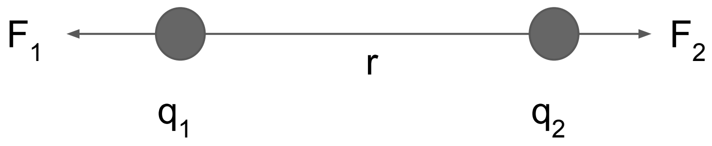
Coulomb law for two charges

The magnitude of the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them.

$$\bar{F}_1 = k_e \frac{q_1 q_2}{r^2} \hat{e}_r \quad \text{Eq 1-1}$$

$$k_e = \frac{1}{4 \pi \epsilon_o} \quad \text{Eq 1-2}$$



q_1	Charge 1
q_2	Charge 2
r	Distance between charges
F_1	Force exerted on charge 1
F_2	Force exerted on charge 2
e_r	Unit vector along direction from q_1 to q_2
k_e	Coulomb constant

Fig 1-1

Force and Field

Let's look differently as Eq 1-1

$$\bar{F}_1 = q_1 \bar{E}_2 \qquad \text{Eq 1-3}$$

$$\bar{E}_2 = \frac{q_2}{4\pi\epsilon_o r^2}\hat{e}_r \qquad \text{Eq 1-4}$$

One can read Eq 1-3 as follows:

The charge q_2 produced a field E_2

The field E_2 exerts a force F_1 on q_1

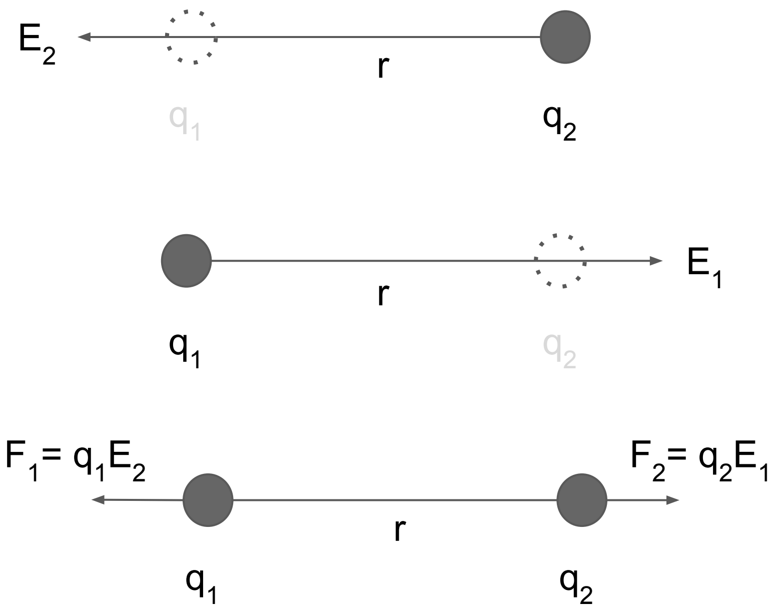


Fig 1-2

Electric field

Hence we can express Coulomb law as follows:

A charge is producing an Electric field in the radial direction **r**.

If a charge q_2 is placed at a location (r , θ), where q_1 is the origin, it will experience a force $\mathbf{F} = q_2 \mathbf{E}_1$

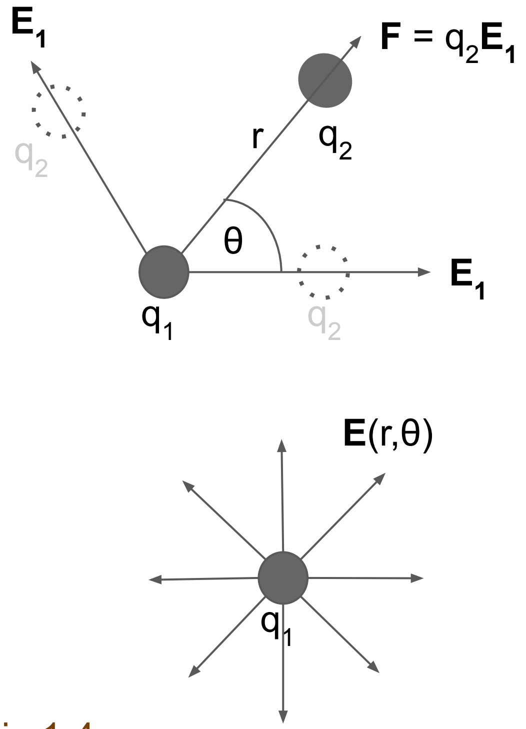


Fig 1-4

The issues with Coulomb's law

The main issue is that we might not know where is q_1 by the time q_2 senses the exerted force $\mathbf{F_2}$ due to charges movement

Let us fix a point in space and observe the field at a specific time instant *t*.

Notice the unit vector $\mathbf{e_r}$' at point P is a retarded form of $\mathbf{e_r}$ (the one at the charge).

$$\hat{e}_r'(t)=\hat{e}_r(t-r'/c)$$ Eq 1-5

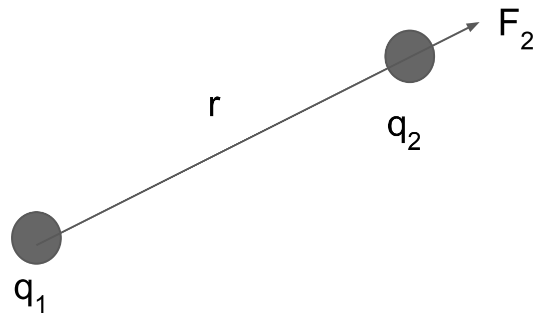


Fig 1-4

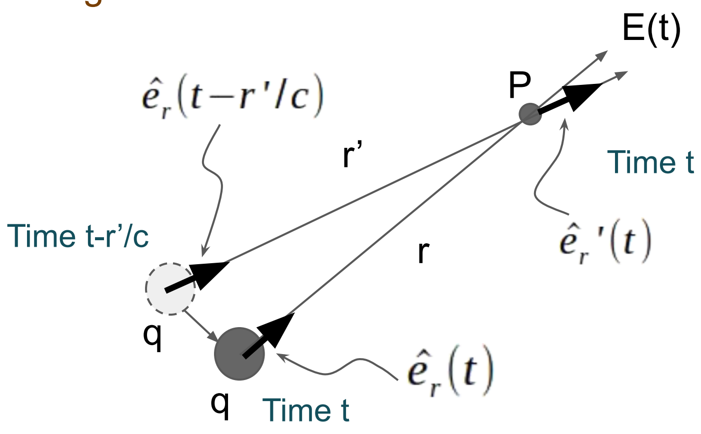


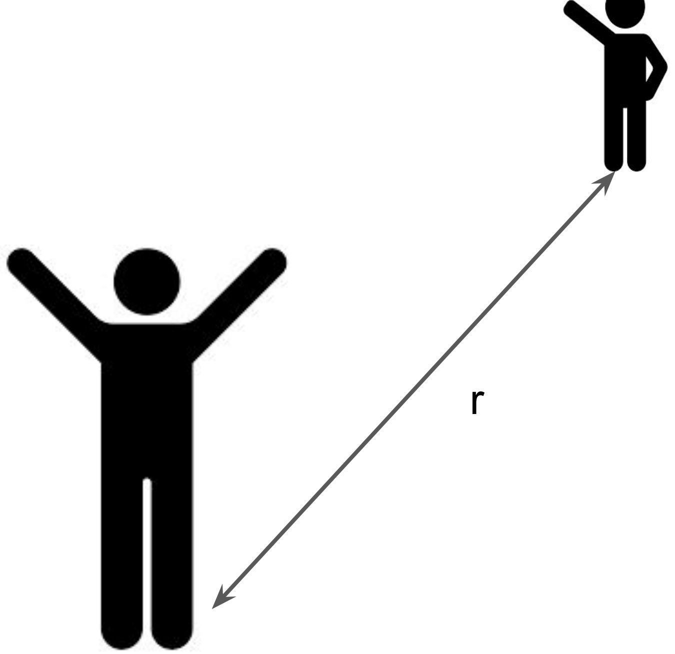
Fig 1-5

What is “Now”

Actually, one can build a philosophical argument about “What is now?”.

Imagine this, you and your friend are setting at the far ends of the lecture room. Do you see your friend at the same time instant you are in or you see him in the past?

You do observe his status earlier by the time it takes for light to travel from him to you (r/c)



Electric field of a moving charge

Hence, for non-static charges, the electric field is affected by the motion of q.

To get an impression of this effect on the generated electric field E(t), Feynman* has presented a mathematical equation that appears to be a form of correction to Coulomb's law

$$\bar{E}(t) = \frac{-q}{4\pi\epsilon_o}\left(\frac{\hat{e}_r'}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{\hat{e}_r'}{r'^2}\right) + \frac{1}{c^2}\frac{d^2}{dt^2}(\hat{e}_r')\right) \quad \text{Eq 1-6}$$

Electrostatic Coulomb's law:

$$\bar{E}(t) = \frac{-q}{4\pi\epsilon_o r'^2}\hat{e}_r'$$

$+$

Apparent Correction terms:

$$\frac{-q}{4\pi\epsilon_o}\left(\frac{r'}{c}\frac{d}{dt}\left(\frac{\hat{e}_r'}{r'^2}\right)\right) + \frac{-q}{4\pi\epsilon_o}\left(\frac{1}{c^2}\frac{d^2}{dt^2}(\hat{e}_r')\right)$$

*The Feynman Lectures on Physics, Volume I, Ch 28

Travelling field

Here, one focuses on a field that can travel for relatively large distance (large r').

Careful observation of Eq 1-6 one notice that the first two terms vary with $1/r'^3$ and $1/r'^2$ respectively (remember $\mathbf{e_r}' = \mathbf{r}'/r'$) while the last time depends on $1/r'$.

Hence for large values of r' one can safely neglect the effect of the first two terms and approximate the field to be mainly due to the last term.

$$\bar{E}(t) \approx \frac{-q}{4\pi\epsilon_o c^2} \frac{d^2}{dt^2}(\hat{e}_r{}') \qquad \text{Eq 1-7}$$

Time varying unit vector

Let us look back into Fig 1-5. Now, without loss of generality, let us rotate the coordinates such that **r'** and **r** are in x-z plane and **r'** is along z.

At any instant, the unit vector has two components

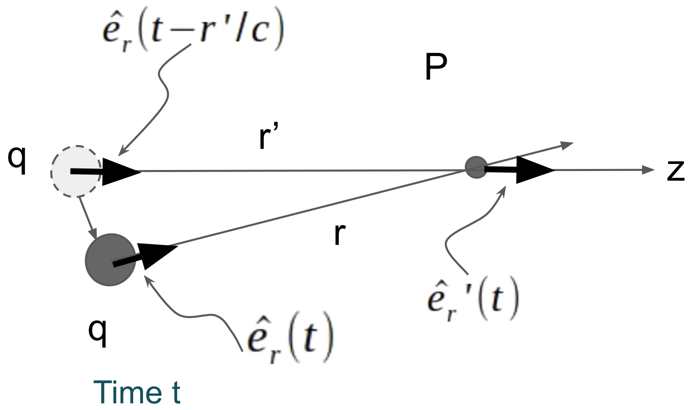


Fig 1-6

$$\hat{e}_r'(t)=e_x'(t)\hat{x}+e_z'(t)\hat{z}$$ Eq 1-8

$$\frac{d^2}{dt^2}\hat{e}_r'(t)=\frac{d^2}{dt^2}\hat{e}_x'(t)\hat{x}+\frac{d^2}{dt^2}\hat{e}_z'(t)\hat{z}$$ Eq 1-9

Restricted motion

Let us set the charge motion is restricted to a region with dimensions much smaller than r'.

Time delay becomes almost constant r/c ~z/c.

$$\frac{d^2}{dt^2}\hat{e}_r'(t)=\frac{d^2}{dt^2}(\frac{x'(t)}{r'})\hat{x}+\frac{d^2}{dt^2}(\frac{z'(t)}{r}')\hat{z}$$

$$\approx\frac{1}{r'}[\frac{d^2}{dt^2}x'(t)\hat{x}+\frac{d^2}{dt^2}z'(t)\hat{z}]$$

Eq 1-10

Using Eq 1-5

$$\frac{d^2}{dt^2}\hat{e}_r'(t)\approx\frac{1}{r'}[\frac{d^2}{dt^2}x(t-r/c)\hat{x}+\frac{d^2}{dt^2}z(t-r/c)\hat{z}]$$

Eq 1-11

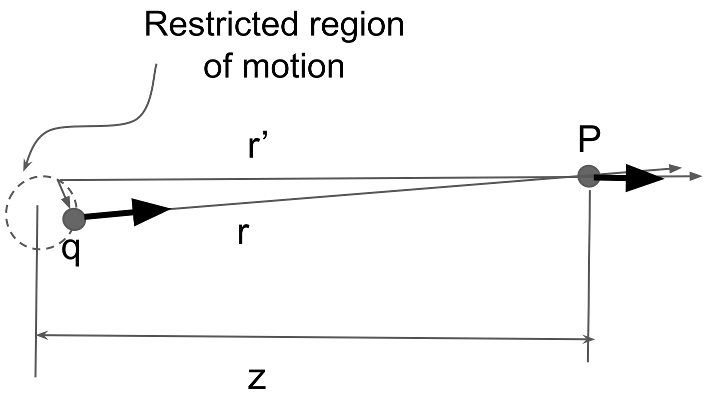


Fig 1-7

Charge acceleration and electric field

We can write Eq 1-11 can be written as

$$\frac{d^2}{dt^2}\hat{e}_r'(t) \approx \frac{1}{r'}[a_x(t-r/c)\hat{x} + a_z(t-r/c)\hat{z}] \qquad \text{Eq 1-12}$$

Here, a_x and a_z are the charge acceleration in x and z directions respectively.

The acceleration along the line of sight, z in our special arrangement, does not contribute to the wiggling of the unit vector. Hence, it can be safe enough to assume that the generated electric field at point P is mainly due to normal acceleration a_x.

The electric field

$$\bar{E}(t) \approx \frac{-q}{4\pi\epsilon_o c^2 r} a_x(t - r/c)\hat{x}$$

Eq 1-13

In Eq 1-13, r’ is approximated by r as in the assumption of restricted region charge movement has little effect on the total distance value. Also, the electric field is along x direction. That is due the special arrangement in our representation. In general it shall be in a direction normal to the direction of sight.

Summary

The most important parts in this discussion is the relations between radiation, field and force.

The charge (static or moving) generate a field.

The field exerts force on another charge placed in space

The observer at point P detects a retarded effect of the charge movement. The delay is r/c.