INTRODUCTION TO OPTICS

Lecture 2: Light propagation

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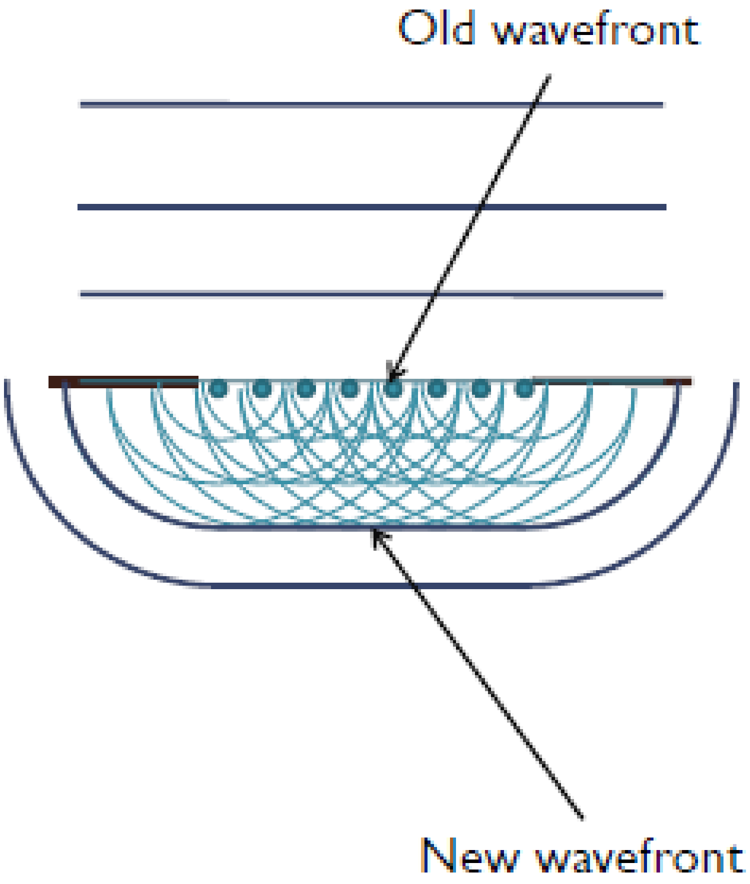
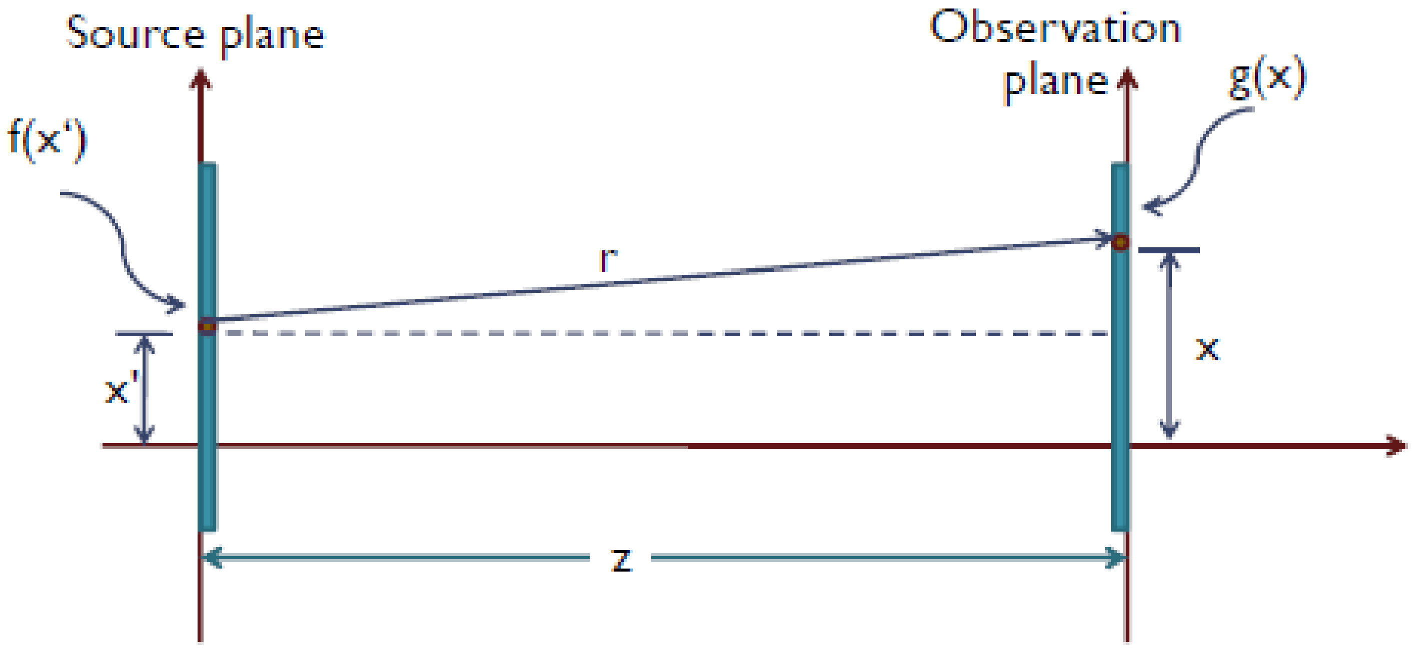
Hygen's principle

- Knowing the wavefront of the light at a certain location, the new wavefront can be reconstructed by combining point sources located on the previous wavefront.



Light propagation

- Knowing the field distribution at a certain location, the field at a distance z can be calculated by dividing the source into point source.



Light propagation

- The source wavefunction is divided into point sources.
- A point source located at point x' has a complex amplitude f(x').
- The wavefunction at a point x on the observation plane due to the point source at x' can be written as

$$g(x;x') = \frac{1}{r}\exp[-j(\omega t - kr)]$$

Diffraction integral

- The field then at point x due to all the points from the source has an integral form.

$$g(x) = \frac{-i}{\lambda} \int_{x'} \frac{1}{r} f(x') \exp[-j(\omega t - kr)] dx'$$

$$r = \sqrt{(x - x')^2 + z^2}$$

- Ignoring fast varying time dependence term

$$g(x) = \int_{x'} \frac{1}{r} f(x') \exp[jkr] dx'$$

http://eosweb.larc.nasa.gov/EDDOCS/Wavelengths_for_Colors.html#blue

Fresnel approximation

- If the distance z is larger that the working window, then Taylor expansion of r is

$$r = z\sqrt{1+\frac{(x-x')^2}{z^2}} = z\left(1+\frac{(x-x')^2}{2z^2}-\frac{(x-x')^4}{8z^4}+\ldots\right)$$

- The phase term in this case is.

$$\phi = kr = \left(kz + k\frac{(x-x')^2}{2z} - k\frac{(x-x')^4}{8z^3}+\ldots\right)$$

Fresnel approximation

- If the third term in the phase expansion is much less than 2π

$$k\frac{\max(x-x')^4}{8z^3} << 2\pi$$

- This term can be ignored and the diffraction equation is written as

$$g(x) \approx \frac{-i}{\lambda}\int_{x'} \frac{1}{z} f(x') \exp[kz + \frac{k(x-x')^2}{2z}] dx'$$

- Note, r in the amplitude is replaced by z as variation in amplitude are not as sensitive as in the phase.

Fresnel propagation

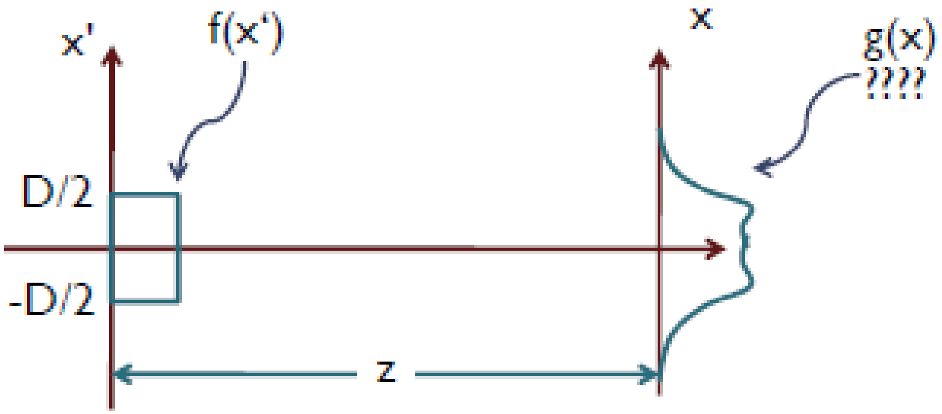
$$g(x) \approx \frac{-i\exp[jkz]}{\lambda z}\int_{x'} f(x')\exp\left[j\frac{k(x-x')^2}{2z}\right]dx'$$

- This integration is referred to as Fresnel propagation (or Fresnel diffraction.)

- It generates the field distribution when the light propagates for a distance z behind a source which has a complex amplitude f(x').

Example 1

- A uniform plane wave is normally incident on an aperture of width D. Write the Fresnel integration at a distance z.



$$g(x) \approx \frac{-i\exp[jkz]}{\lambda z}\int_{-D/2}^{D/2}\exp\left[j\frac{k(x-x')^2}{2z}\right]dx'$$

Exercise 1

- Calculate and plot the diffraction patterns for light when propagating away from a rectangular slit

Fresnel propagation

- Expanding the quadratic term in the phase

$$g(x) \approx \frac{-ie^{jkz}}{\lambda z} \int_{x'} f(x') \exp\left[j \frac{k(x^2 - 2xx' + x'^2)}{2z} \right] dx'$$

$$= \frac{-ie^{jkz}}{\lambda z} e^{j\frac{k}{2z}x^2} \int_{x'} f(x') e^{j\frac{k}{2z}x'^2} e^{-j\frac{k}{z}xx'} dx'$$

- Ignoring the constant phase term k_z

$$g(x) \frac{e^{j\frac{k}{2z}x^2}}{\lambda z} \int_{x'} f(x') e^{j\frac{k}{2z}x'^2} e^{-j2\pi\left(\frac{x}{\lambda z}\right)x'} dx'$$

Fresnel propagation

- We can re-write the Fesnel propagation as

$$g(x) = i\frac{e^{j\frac{k}{2z}x^2}}{\lambda z}\int_{x'}\tilde{f}(x')e^{-j2\pi\nu x'}dx'$$

$$\tilde{f}(x') = f(x')e^{j\frac{k}{2z}x'^2} \quad \text{and} \quad \nu = \frac{x}{\lambda z}$$

- The equation above represents a Fourier transform of a modified function $\tilde{f}(x')$

$$g(\nu) = i\frac{e^{j\pi\lambda z\nu^2}}{\lambda z}F.T\left\{\tilde{f}(x')\right\}_{\nu=\frac{x}{\lambda z}}$$

Fraunhofer approximation

- To obtain a Fourier transform for the source function, the quadratic term in $\tilde{f}(x')$ has to vanish.

- That happens when

$$\frac{k\max(x')^2}{2z} << 2\pi$$

- In this case we obtain the Fourier transform of the source

$$g(\nu) = i\frac{e^{j\pi\lambda z\nu^2}}{\lambda z} F.T\{f(x')\}_{\nu=\frac{x}{\lambda z}}$$

Exercise 2

- Calculate the Fraunhofer diffraction for a rectangular slit of width 1 mm at a distance 1 m away when the source is a read laser of wavelength 633 nm