Fundamentals of Optics

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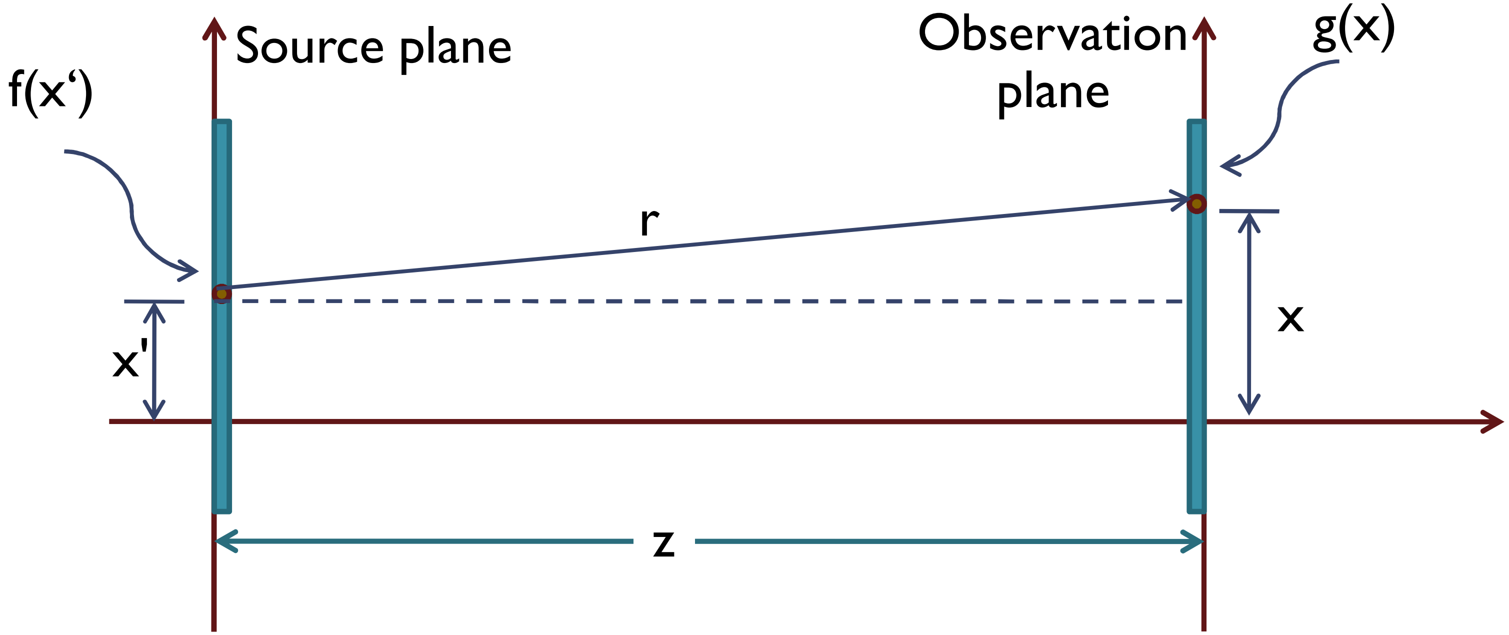
FOURIER OPTICS

Light propagation

- When distance z is large enough



- The field at the observation plane is

$$g(\nu) = i\,\frac{e^{j\pi\lambda z \nu^2}}{\lambda z}\,F.T\{f(x')\}_{\nu=\frac{x}{\lambda z}}$$

Fourier transform

- Fourier transformation of the source is observed at a distance z.

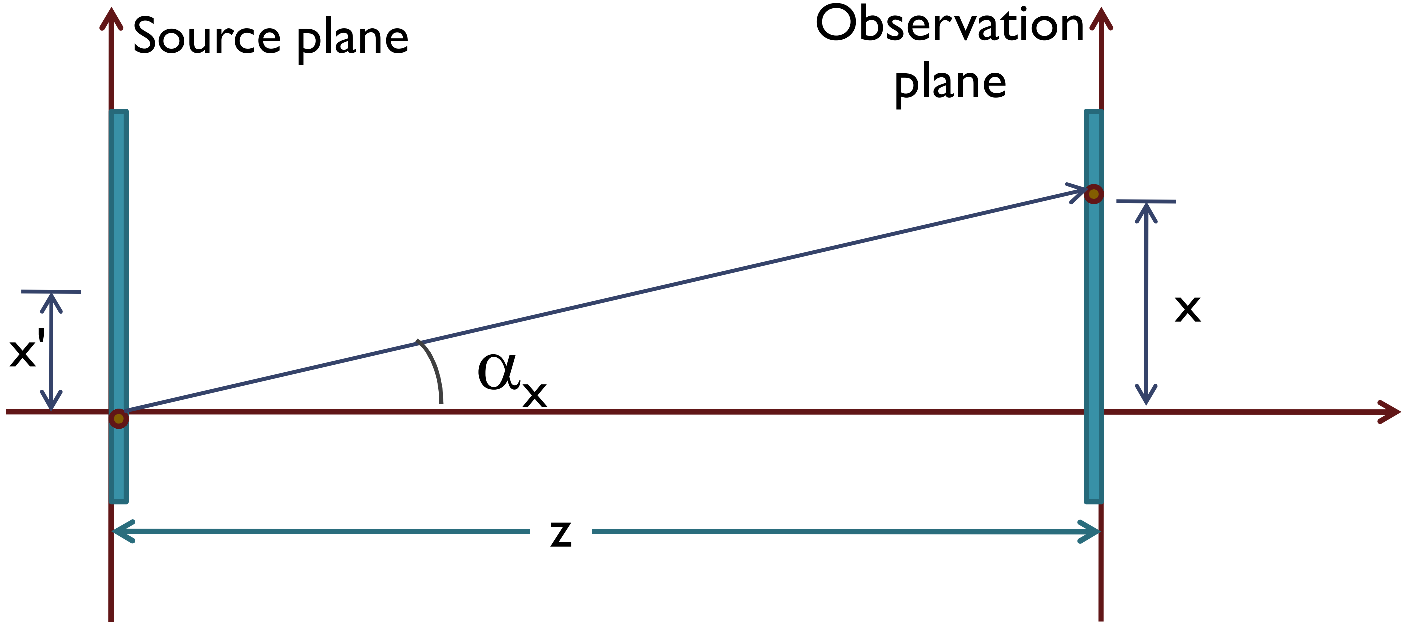
$$z >> \frac{k \max(x')^2}{4\pi}$$

- The frequency v is related to the location x

$$\nu = \frac{x}{\lambda z}$$

- The frequency v has units of 1/m and it is referred to as spatial-frequency

Spatial frequency



$$\nu = \frac{x}{\lambda z} = \frac{1}{\lambda}\tan\alpha \quad \text{if } \alpha \text{ is very samll then } \tan\alpha \approx \alpha$$

$$\nu \approx \frac{\alpha}{\lambda}$$

Optical Fourier transform

- The Fourier transform is then

$$F.T\{f(x')\} = \int_{-\infty}^{\infty} f(x')e^{-j2\pi\nu x'}dx' = \int_{-\infty}^{\infty} f(x')e^{-j2\pi\frac{\alpha}{\lambda}x'}dx'$$

- The phase term is

$$\phi = \frac{2\pi}{\lambda}\alpha_x x' = k\alpha_x x'$$

- Remember from the wave-vector definition

$$\vec{k} = k(\alpha_x, \alpha_y, \alpha_z) = (k_x, k_y, k_z)$$

$$\text{Hence,} \quad \phi = k_x x'$$

Optical Fourier transform

- The Fourier transform is then

$$F(k_x) = F.T.\{f(x')\} = \int_{-\infty}^{\infty} f(x')e^{-jk_x x'}dx'$$

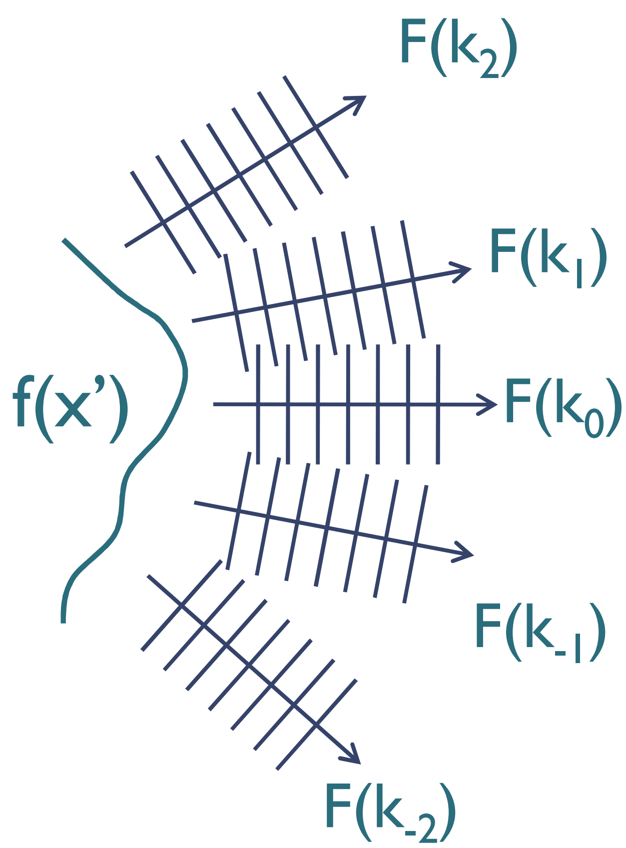
- The inverse Fourier transform is

$$f(x') = F.T.^{-1}\{F(k_x)\} = \int_{-\infty}^{\infty} F(k_x)e^{jk_x x'}dk_x$$

- The exponent term represents a plane wave in a direction $(k_x,0,k_z)$, where $k_z \cong k$

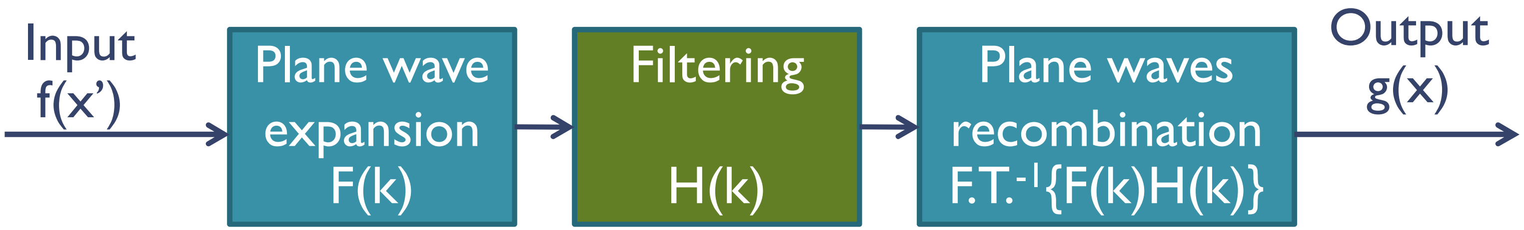
Plane-wave expansion

- Any function f(x') can be presented as an infinite summation of plane waves ($e^{j(k_x x' + k_z z)}$) each propagates in a different direction ($\vec{k}$) and amplitude (F(k)).



Fourier Optics

- Basics of Fourier Optics
  - Any field can be expanded into an infinite summation of plane waves.
  - Manipulating each plane wave for specific task. This process is referred to as *filtering*.
  - Recombining the resultant plane waves to reconstruct the output field.



- The question is how to manipulate the waves?

Back to Fresnel

- Fresnel propagation gives

$$g(x)\frac{e^{j\frac{k}{2z}x^2}}{\lambda z}\int_{x'} f(x')e^{j\frac{k}{2z}x'^2}e^{-j2\pi\left(\frac{x}{\lambda z}\right)x'}dx'$$

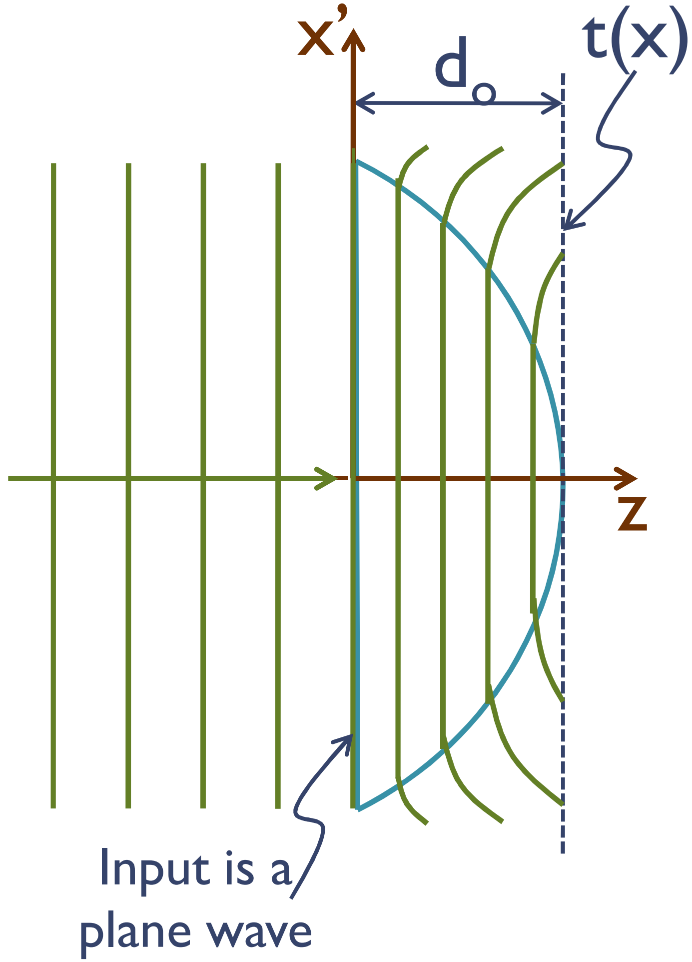
- One way to remove the phase $e^{j\frac{k}{2z}x'^2}$ is to have z very large (Fraunhofer).
- Another way is to introduce an element that has transmittance of

$$t(x') = e^{-j\frac{k}{2z}x'^2}$$

Spherical lens and Fourier optics

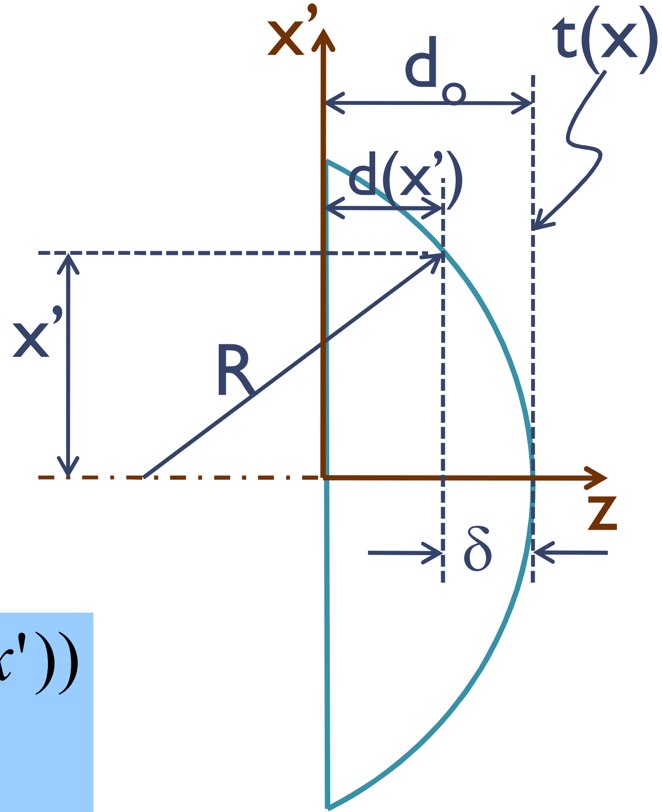
- Consider a spherical lens of radius R and thickness d_o and refractive index n.
- The light at the output of the lens gains phase ϕ(x') due to the change of the delay with x'.

$$t(x') = e^{j\phi(x')}$$



Lens phase

- Light passing by a point x' experience a delay over distance d(x') inside the lens with refractive index n and another delay over a distance d_o-d(x) in air.



$$\phi(x') = knd(x') + k(d_o - d(x'))$$
$$= k(n-1)d(x') + kd_o$$
$$d(x') = d_o - \delta$$
$$\delta = R - \sqrt{R^2 - x'^2}$$

Lens phase

- Using Fresnel approximation

$$\sqrt{R^2 - x'^2} \approx R - \frac{x'^2}{2R}$$

- Hence

$$\delta \approx R - (R - \frac{x'^2}{2R}) = \frac{x'^2}{2R}$$

$$d(x') = d_o - \delta = d_o - \frac{x'^2}{2R}$$

$$\phi(x') = k(n-1)d(x') + kd_o = k(n-1)\left(d_o - \frac{x'^2}{2R} \right) + kd_o$$

Lens phase

- The lens phase is then

$$\phi(x') = \phi(x') = -k(n-1)\frac{x'^2}{2R} + knd_o$$

- Ignoring the constant phase term

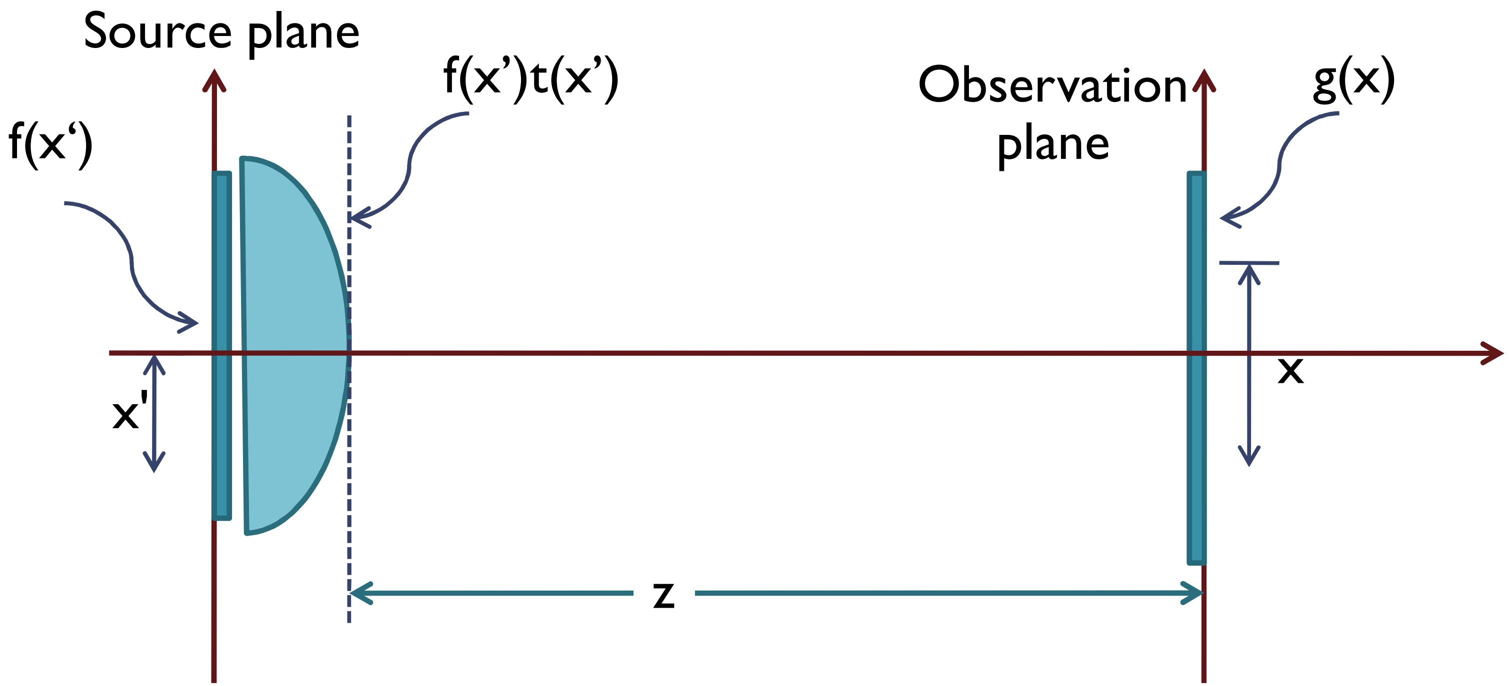
$$\phi(x') = \phi(x') = -k\frac{x'^2}{2f}, \text{ where } f = \frac{R}{n-1}$$

- The lens transmittance t(x') is

$$t(x') = e^{-jk\frac{x'^2}{2f}}$$

- Which is the needed term when z = f

Fourier transform lens



$$g(x)=\frac{e^{j\frac{k}{2z}x^2}}{\lambda z}\int_{x'} f(x')t(x')e^{j\frac{k}{2z}x'^2}\,e^{-j2\pi\left(\frac{x}{\lambda z}\right)x'}dx'$$

Fourier transform lens

- The Fresnel propagation is then

$$g(x) = \frac{e^{j\frac{k}{2z}x^2}}{\lambda z} \int_{x'} f(x') e^{-j\frac{k}{2f}x'^2} e^{j\frac{k}{2z}x'^2} e^{-j2\pi\left(\frac{x}{\lambda z}\right)x'} dx'$$

$$= \frac{e^{j\frac{k}{2z}x^2}}{\lambda z} \int_{x'} f(x') e^{j\left(\frac{1}{z}-\frac{1}{f}\right)\frac{k}{2}x'^2} e^{-j2\pi\left(\frac{x}{\lambda z}\right)x'} dx'$$

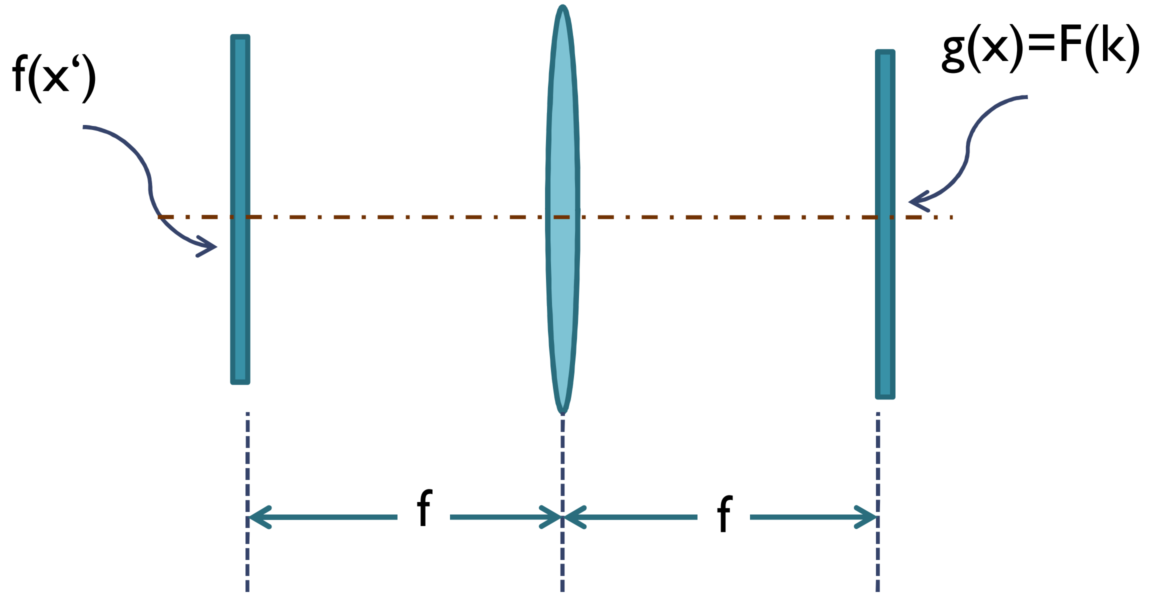
- If z = f

$$g(x) = \frac{e^{j\frac{k}{2f}x^2}}{\lambda f} \int_{x'} f(x') e^{-j2\pi\left(\frac{x}{\lambda f}\right)x'} dx' = \frac{e^{j\frac{k}{2f}x^2}}{\lambda f} F.T.\{f(x')\}_{v=\frac{x}{\lambda f}}$$

Fourier transform lens

- Lens cancels out the quadratic phase term in the Fresnel propagation at $z = f$.
- The lens brings Fraunhofer domain to the back focal plane.
- The amplitude of the field is reduced by a factor of $1/f$ compared to $1/z$ in Fraunhofer case.
- The spatial frequency, *v*, is proportional to $1/f$ instead of $1/z$ (shrinks the space).

Optical Fourier transform

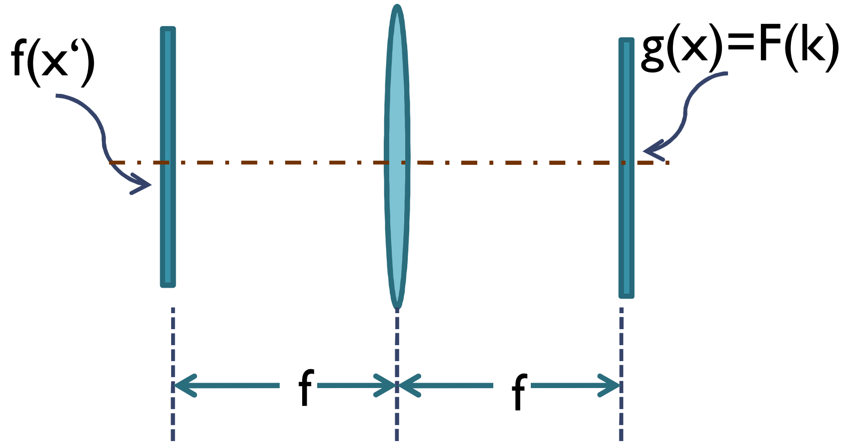


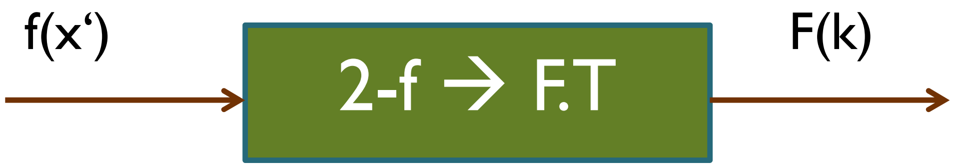
- If the source is places at a distance f infront of the mirror the phase term outside the integration will cancel out

$$g(x) = \frac{1}{\lambda f}\int_{x'} f(x')e^{-j2\pi\left(\frac{x}{\lambda f}\right)x'}dx' = \frac{1}{\lambda f}F.T.\{f(x')\}_{v=\frac{x}{\lambda f}}$$

2-f system

- This configuration is referred to as 2-f system.



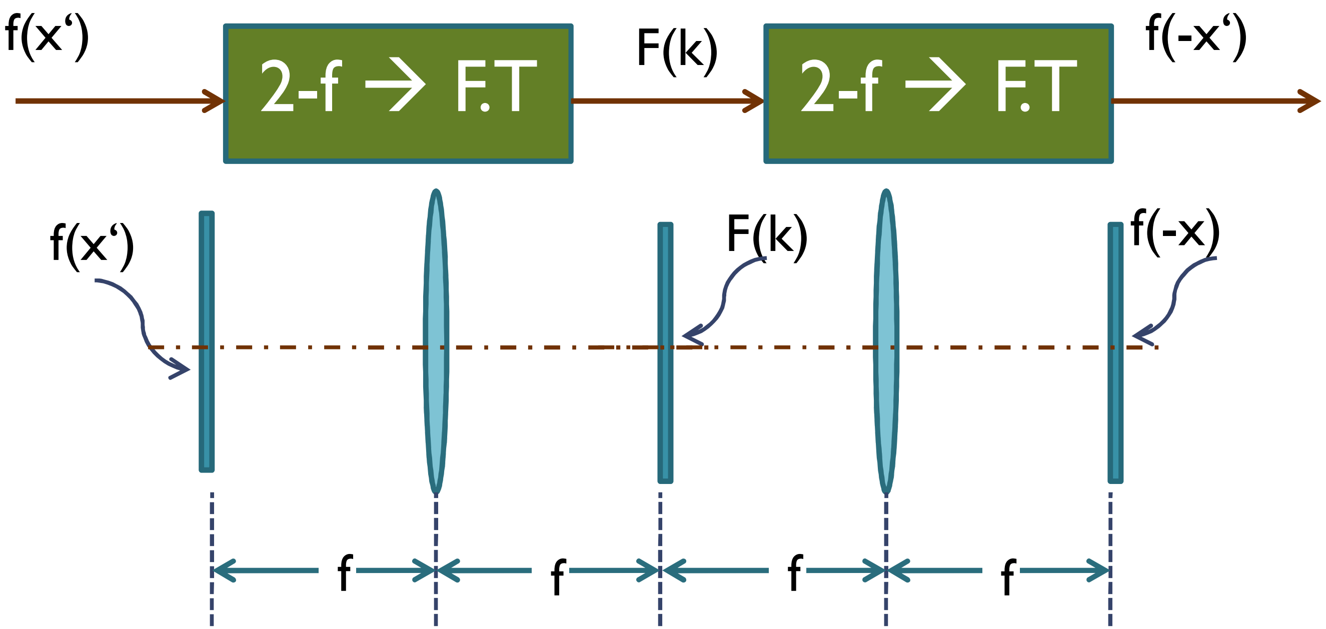


- This is the building block for any Fourier optic system.

4-f system

- From Fourier transform properties

$$F(k) = F.T.\{f(x')\} \rightarrow F.T.\{F(k)\} = f(-x')$$



- This configuration is referred to as 4-f system.

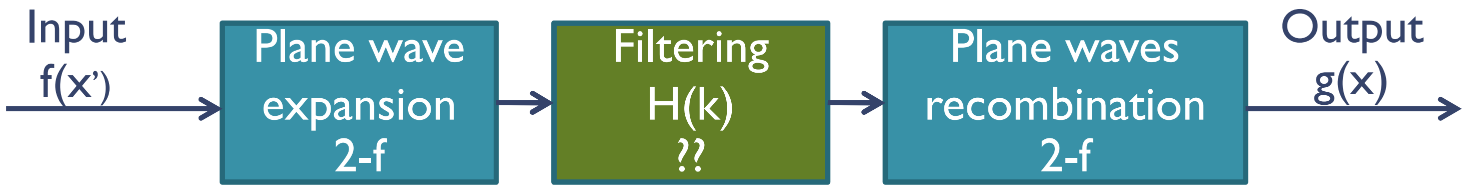
Fourier optics

- In 2-f system, the output is an exact Fourier transform of the input

- In 4-f system the output is the same as the input (but flipped).
- The second 2-f system reconstructs the output from the Fourier transform

Fourier optics

- The basics of Fourier optics

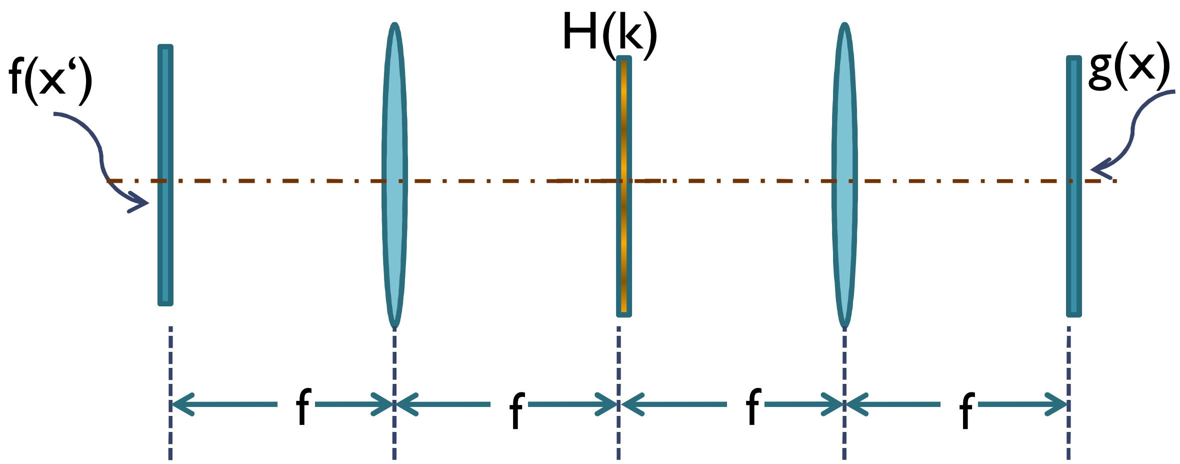


- The field at the back focal plane of the first lens is th F.T. of the source.
- Frequency v is proportional to space.

$$\nu = \frac{x}{\lambda f} = \frac{1}{\lambda}\frac{x}{f} = \frac{\alpha}{\lambda} = \frac{k_x}{2\pi} \rightarrow x = \frac{\lambda f k_x}{2\pi}$$

- The frequency components of the source can be manipulated by placing an element H(k).

Fourier optics system



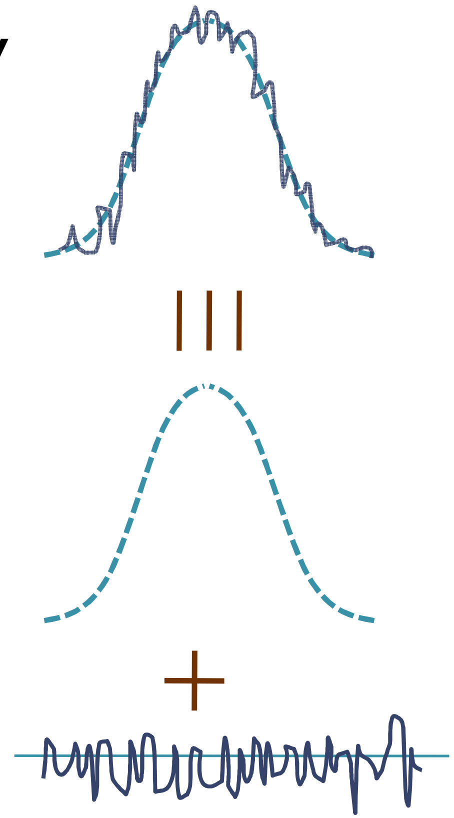
$$g(x) = F.T.\{F(k)H(k)\}$$

Example: beam filtering

- In optics experiment, laser source usually produce noisy beam profile.
- A noisy Gaussian beam can be written as

$$f(x) = e^{-\frac{x^2}{2w^2}} + f_n(x)$$

- fn(x) is a rapidly varying noise function.
- Simple example is sin function with high frequency

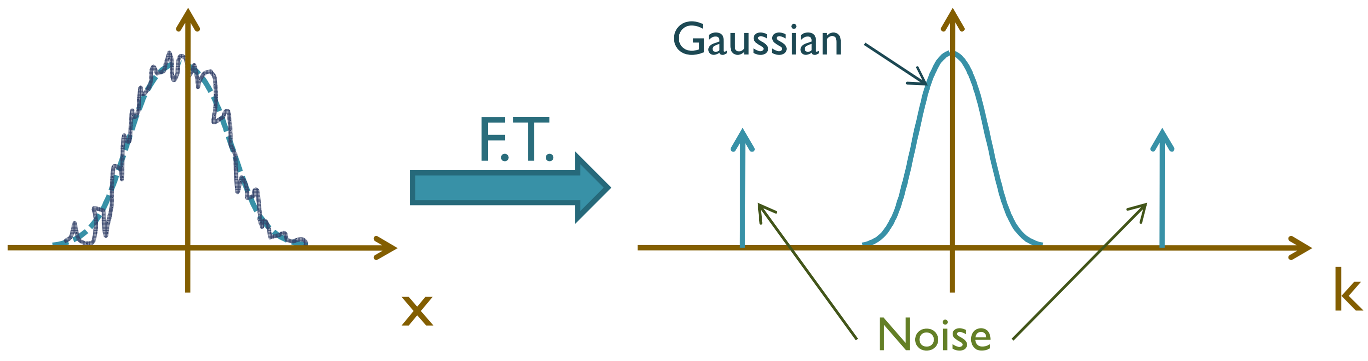


Example: beam filtering

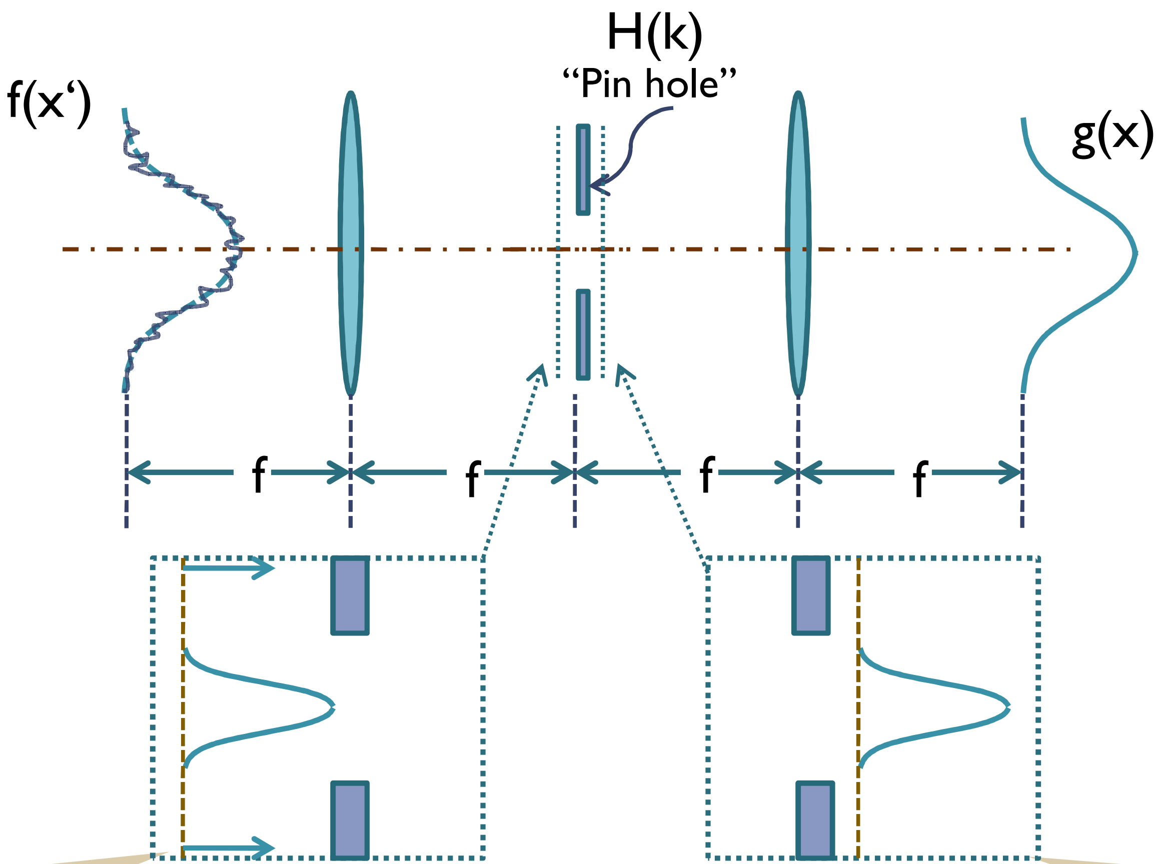
- Noise corresponds to rapid variation and hence high frequency components.
- Gaussian beam corresponds to slow variation and hence low frequency components.

$$f(x) = e^{-\frac{x^2}{2w^2}} + \cos(2\pi v_o x)$$

$$F(k) = F.T.\{f(x)\} = e^{-2k^2w^2} + \tfrac{1}{2}\delta(k - v_o) + \tfrac{1}{2}\delta(k + v_o)$$



Example: beam filtering



At the focal plane F(k) has low frequency component "Gaussian" and high frequency components "noise"

H(k) filters out the high frequency components "noise." The output is $F(k)H(k)=\exp[-2k2w^2]$