Fundamentals of Optics

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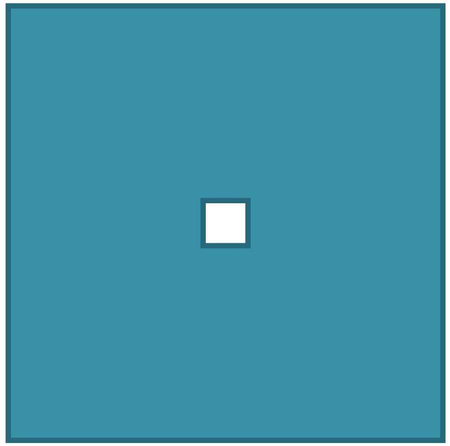
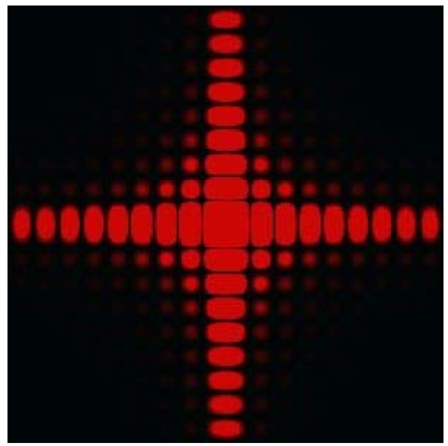
DIFFRACTION

The concept

- Diffraction is normally taken to refer to various phenomena which occur when a wave encounters an obstacle. It is described as the apparent bending of waves around small obstacles and the spreading out of waves past small openings.

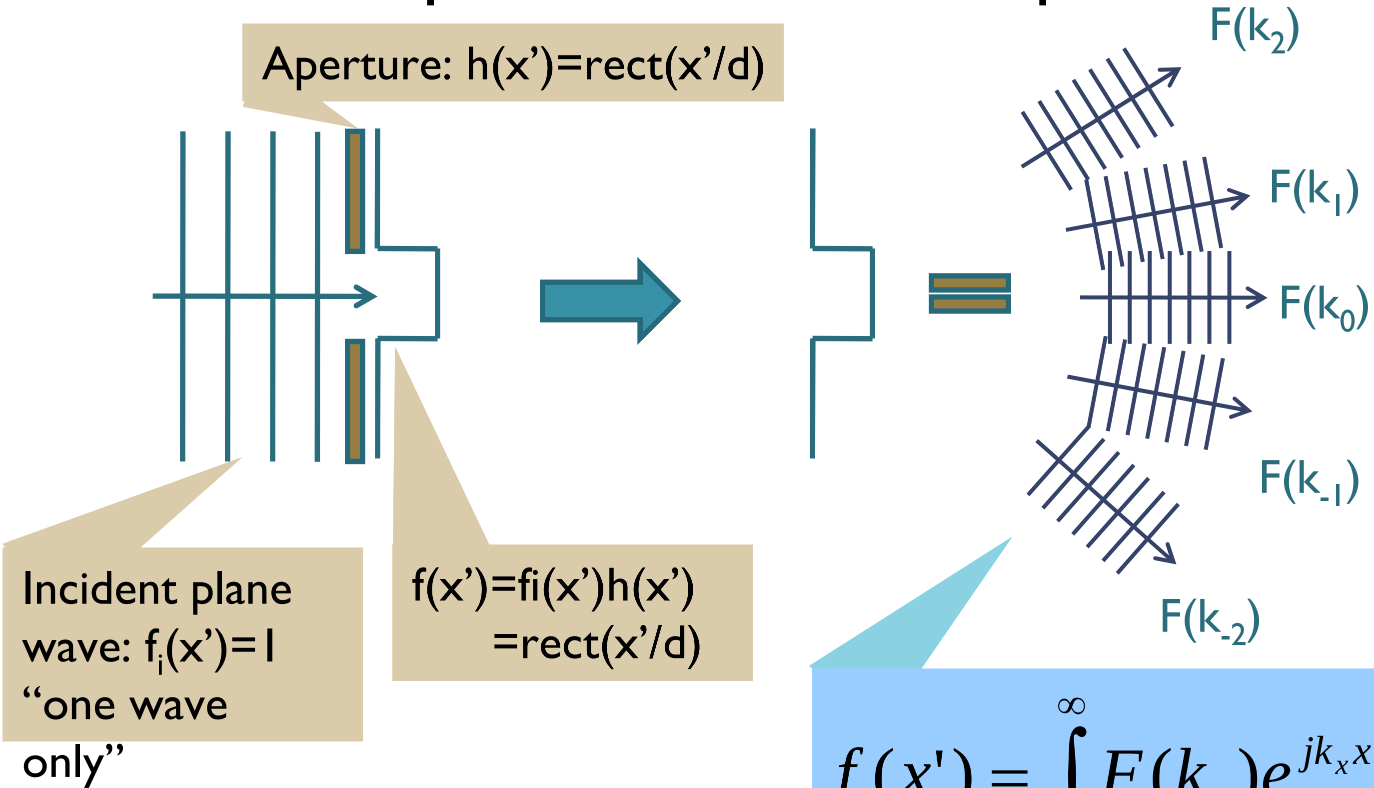




http://en.wikipedia.org/wiki/Diffraction

Diffraction and Fourier optics

- Diffraction can be explained using the plane-wave expansion in Fourier optics.



$$f(x') = \int_{-\infty}^{\infty} F(k_x) e^{jk_x x'} dk_x$$

The concept of diffraction

- The originally incident light has one plane-wave ,or frequency component only (k_x=0).
- The aperture changed the amplitude of the incident wave to a rect function.
- The plane-wave expansion of the rect function yields to more frequency components.

$$F(k_x) = F.T.\{rect(x/d)\} = \sin(k_x d)/k_x$$

- The aperture have added higher order frequency components the incident wave, and hence the light will spread (diverge).

Diffraction pattern

- At z=0

$$f(x';z=0)=\int_{-\infty}^{\infty}F(k_x)e^{jk_xx'}dk_x$$

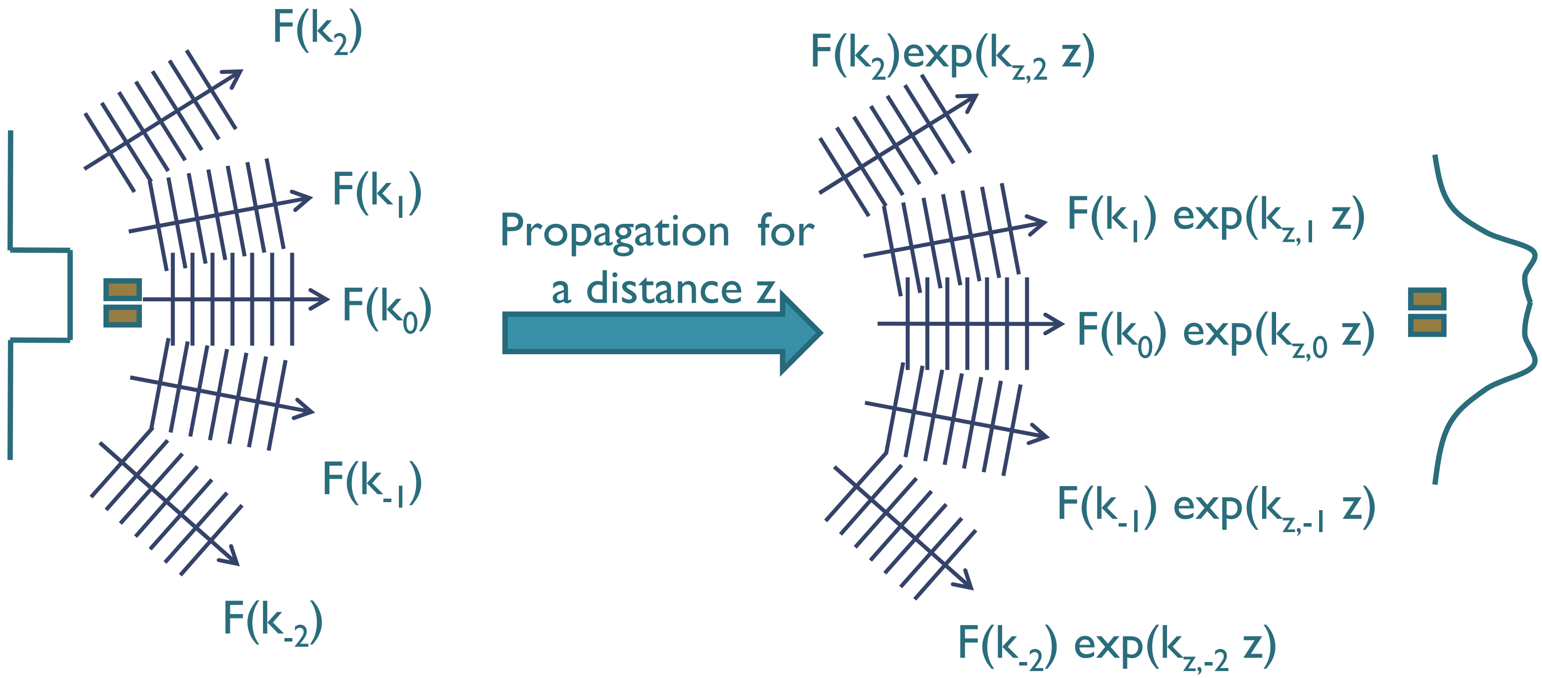
- Each plane wave travels in a certain direction $\vec{k}=(k_x,k_z)$ with an amplitude (F(*k*)).
- At a distance z

$$f(x';z)=\int_{-\infty}^{\infty}F(k_x)e^{jk_xx'}e^{jk_zz}dk_x$$

- Remember

$$k_z=\sqrt{k^2-k_x^2}\approx k-\frac{k_x^2}{2k}$$

Diffraction pattern



$$f(x;z) = e^{jkz} \int_{-\infty}^{\infty} F(k_x) e^{j\frac{k_x^2}{2k}z} e^{jk_x x} dk_x$$

- This is the same Fresnel propagation written in terms of $F(k_x)$

Summery of the concept

- Again, diffraction results from light passing though an aperture or obstacle.
- The aperture(or obstacle) adds higher order frequencies to the light.
- Each frequency has an amplitude and propagates in different direction, hence gain different phase.
- The diffraction pattern observed at a distance z is the summation of all these plane waves with their different phases and amplitudes.

Diffraction pattern and far field

- Usually diffraction pattern is referred to the far field intensity, or z is very large.

- That is t he case of Fraunhofer propagation.

- The diffraction pattern is proportional to the Fourier transform of the aperture

$$f_{diff}(x) \propto F(k_x)\Big|_{k_x = \frac{2\pi x}{\lambda z}}$$

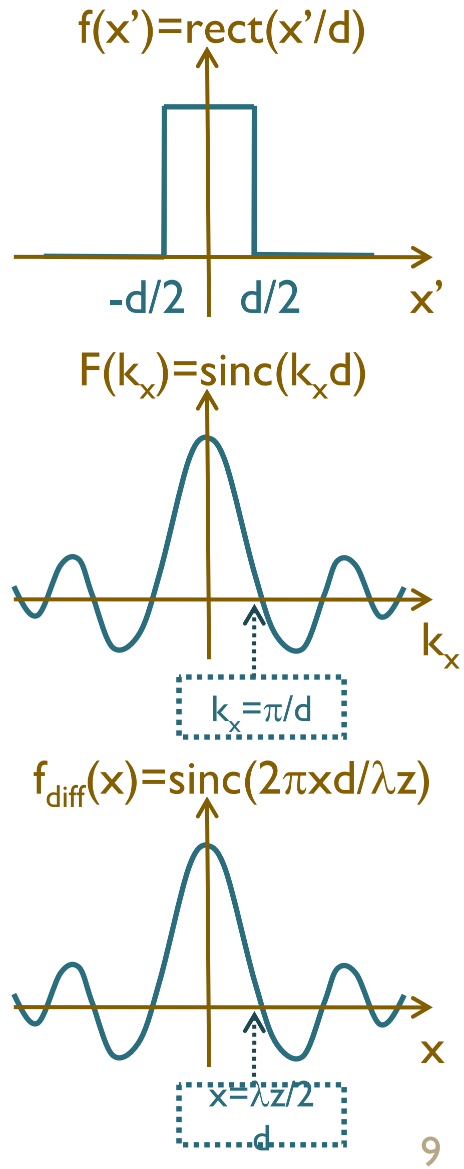
Single slit diffraction

- For the case of a single slit, the transmittance of the aperture is rect function.

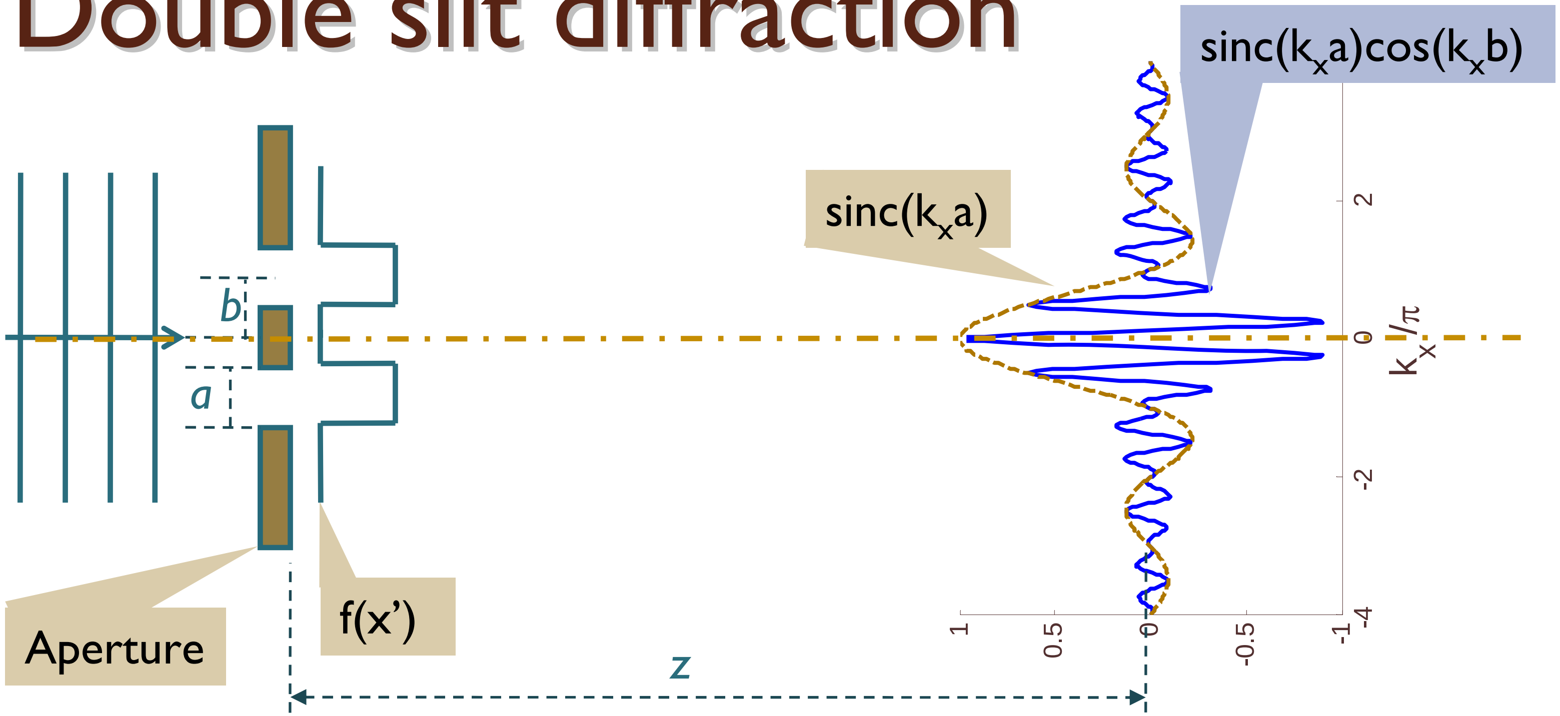
$$F(k_x) = F.T.\left\{ rect\left(\frac{x}{d} \right) \right\} = d\,\frac{\sin(k_x d)}{k_x d}$$

- The diffraction pattern is then

$$f_{diff}(x) \propto \lambda z \frac{\sin\left(2\pi d \dfrac{x}{\lambda z}\right)}{2\pi x}$$



Double slit diffraction



$$f(x') = \text{rect}\left(\frac{x-b}{a}\right) + \text{rect}\left(\frac{x+b}{a}\right)$$

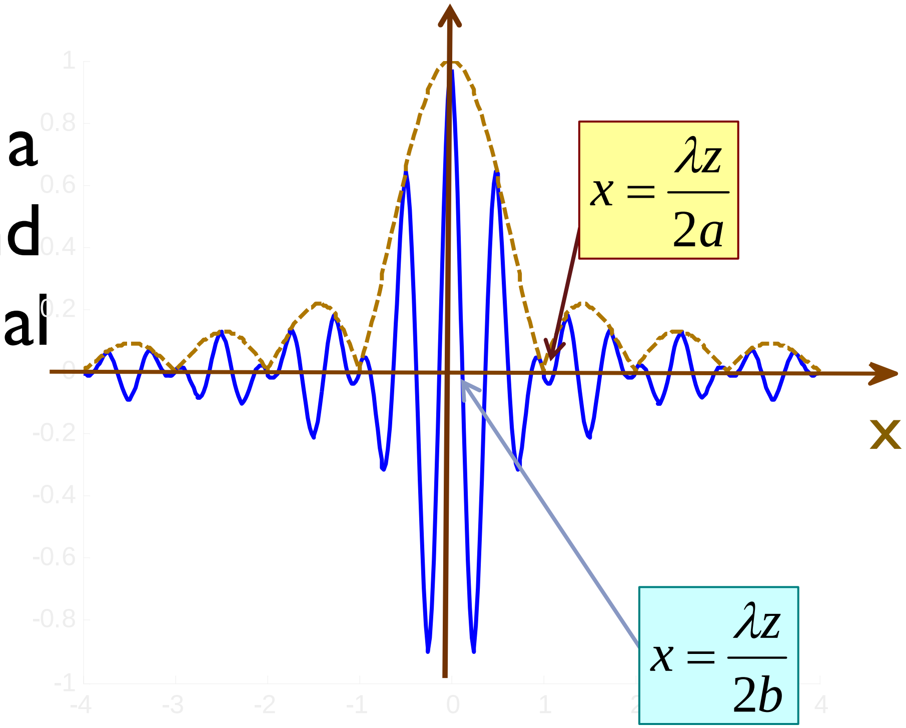
$$F(k_x) = a\text{sinc}(k_x a)e^{-jk_x b} + a\text{sinc}(k_x a)e^{+jk_x b}$$
$$= 2a\text{sinc}(k_x a)\cos(k_x b)$$

Double slit diffraction

- The diffraction pattern is

$$f_{diff}(x) = 2a\,\mathrm{sinc}\left(\frac{2\pi x}{\lambda z}a\right)\cos\left(\frac{2\pi x}{\lambda z}b\right)$$

- The pattern has a sinc envelope and a fast cosinusoidal term.



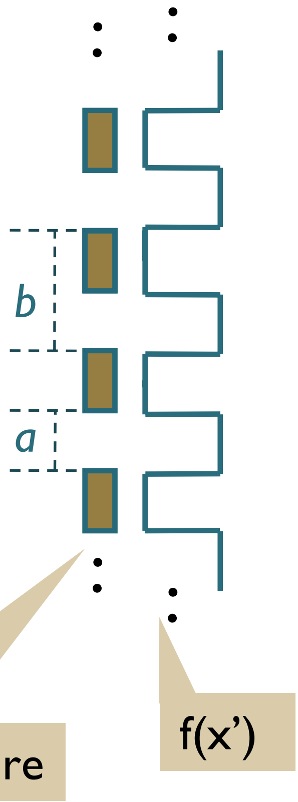
Periodical slits

- For an aperture of repeated slits

$$f(x') = \cdots + \mathrm{rect}\left(\frac{x'+b}{a}\right) + \mathrm{rect}\left(\frac{x'}{a}\right) + \mathrm{rect}\left(\frac{x'-b}{a}\right) + \cdots$$

- F(x') can be written in a form of convolution

$$f(x') = \mathrm{rect}\left(\frac{x'}{a}\right) \otimes comb\left(\frac{x'}{b}\right)$$

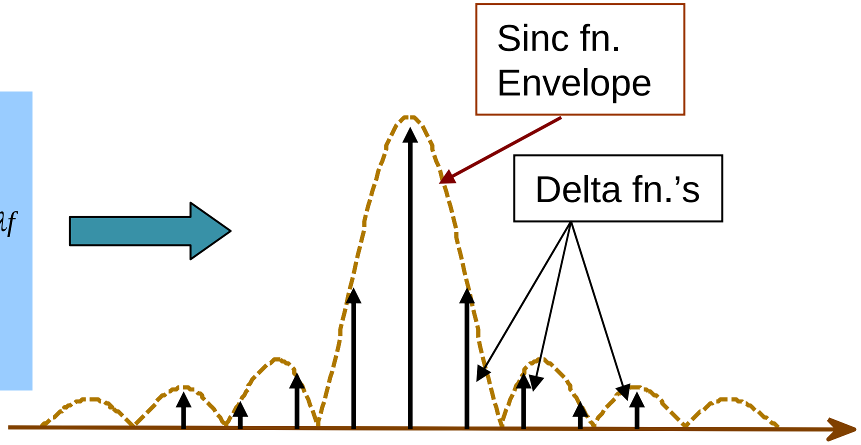


$\mathrm{rect}\left(\frac{x'}{a}\right)$ $comb\left(\frac{x'}{b}\right)$

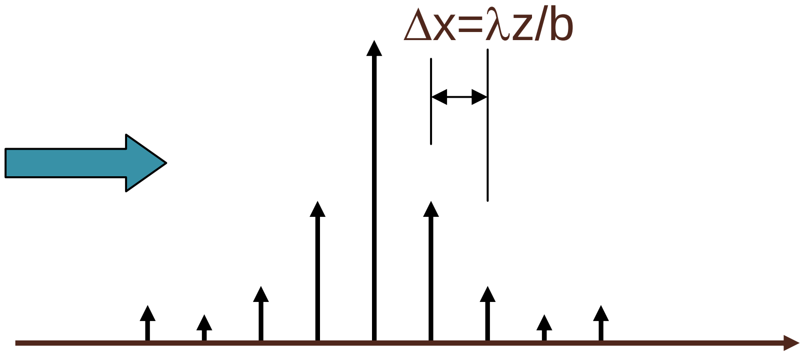
Far field by periodical slits

$$g(x) = F.T.\left\{ \mathrm{rect}\left(\frac{x'}{a}\right) \otimes comb\left(\frac{x'}{b}\right) \right\}_{v=x/\lambda f}$$

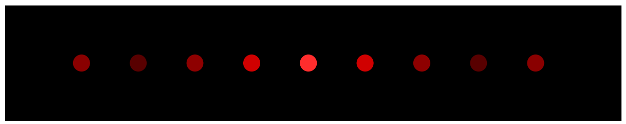
$$= \mathrm{sinc}\left(\frac{\pi x a}{\lambda z}\right) \times comb\left(\frac{x b}{\lambda z}\right)$$



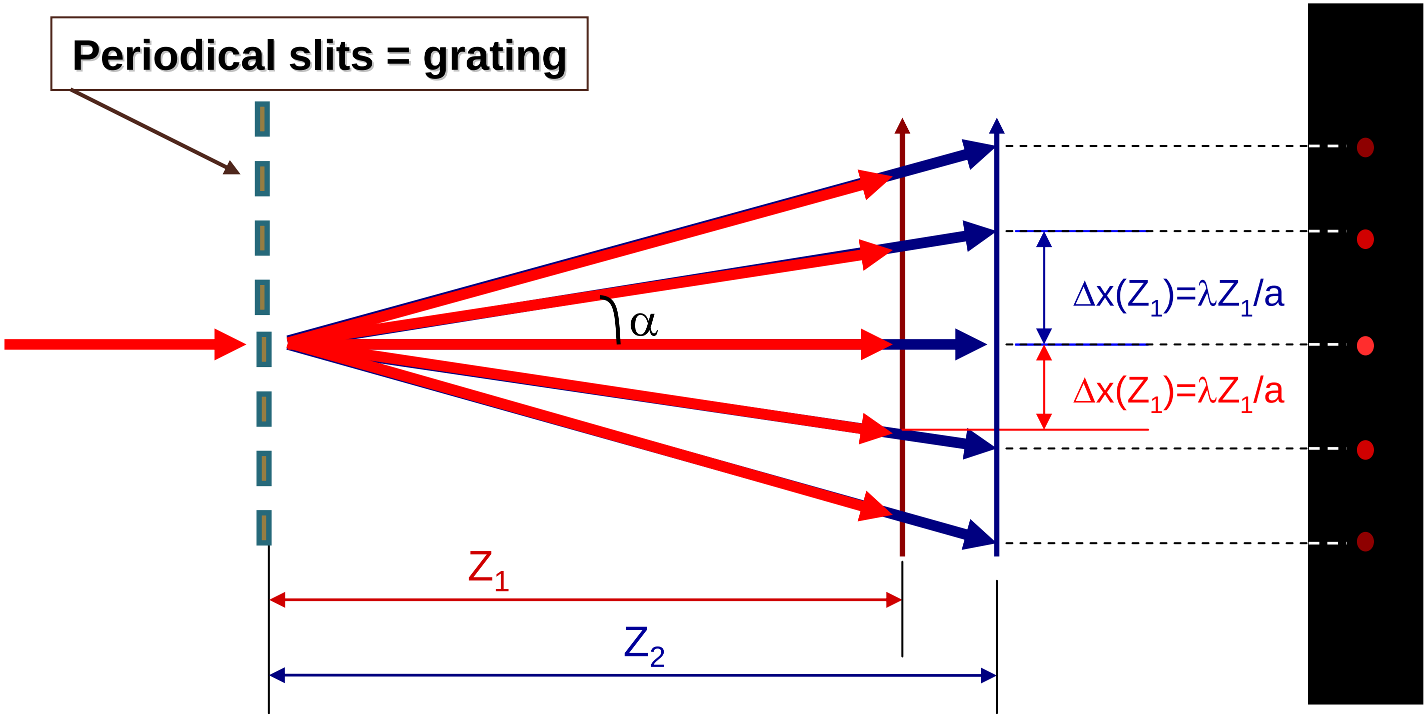
Removing the envelope



What we actually see on the screen



Diffraction representation

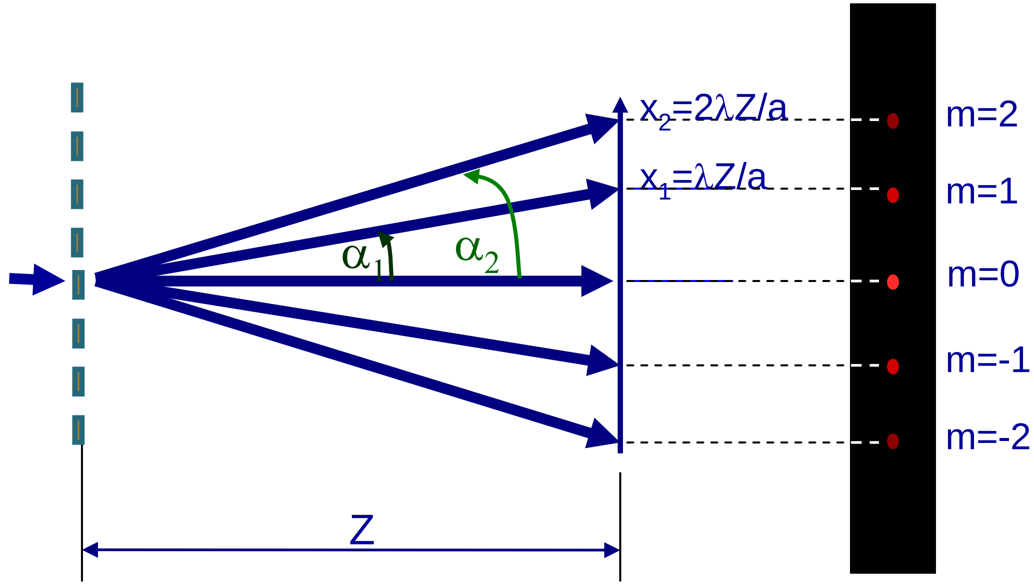


The spacing between the spots (delta functions) Δx at any location Z behind the grating is

$$\Delta x(Z) = \frac{\lambda Z}{a}, \quad \tan\alpha = \frac{\Delta x(Z)}{Z} = \frac{\lambda}{a}$$

Diffraction grating

The Grating diffracts the incident light into several directions each with angle α_m.



Each spot (called diffraction order) is located at a distance x_m.

$$x_m = m\frac{Z\lambda}{a}, \qquad \tan\alpha_m = \frac{x_m}{Z} = m\frac{\lambda}{a}$$

Diffraction orders

- At small angles approximation.

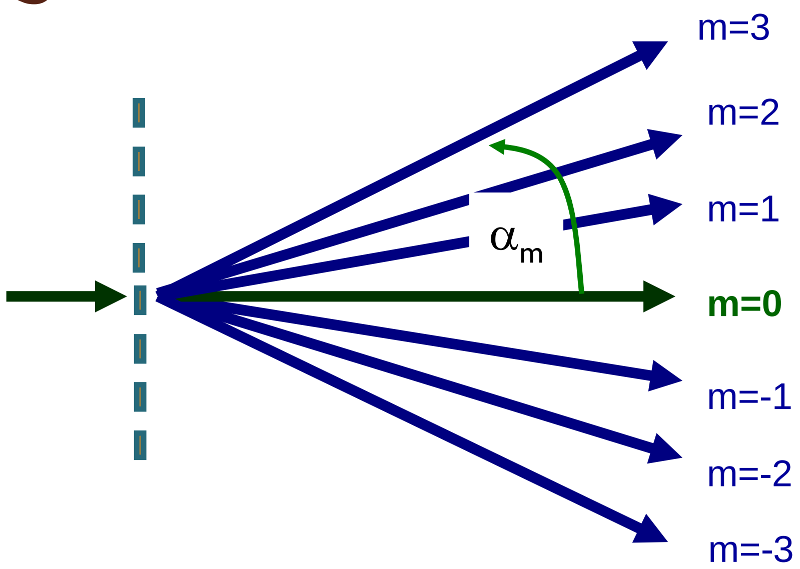
$$\tan\alpha_m \approx \alpha_m = \sin\alpha_m = m\frac{\lambda}{a}$$

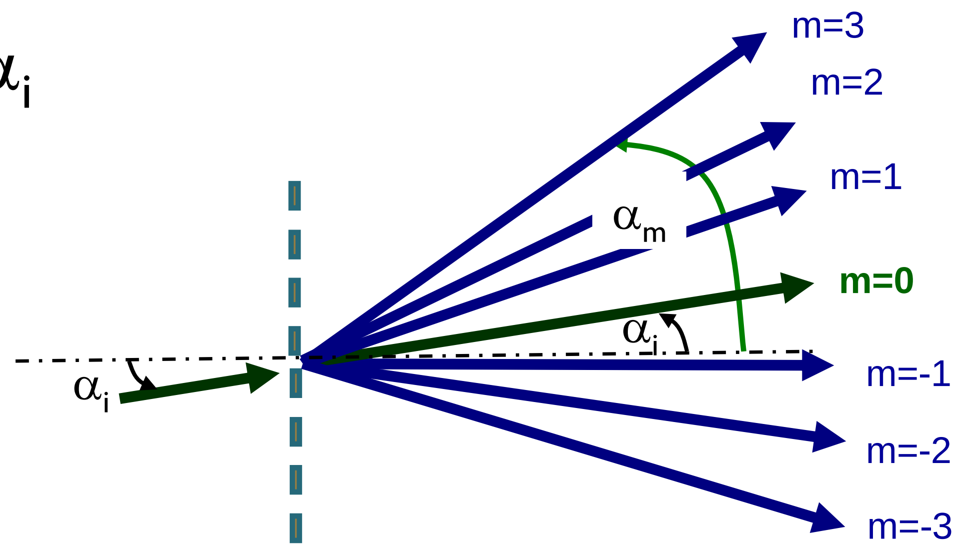
- When the incident light has an angle α_i

$$\alpha_m = \alpha_i + m\frac{\lambda}{a}$$

Or

$$\sin\alpha_m = \sin\alpha_i + m\frac{\lambda}{a}$$





The grating equation

- This last equation is

$$\sin\alpha_m = \sin\alpha_i + m\frac{\lambda}{a}$$

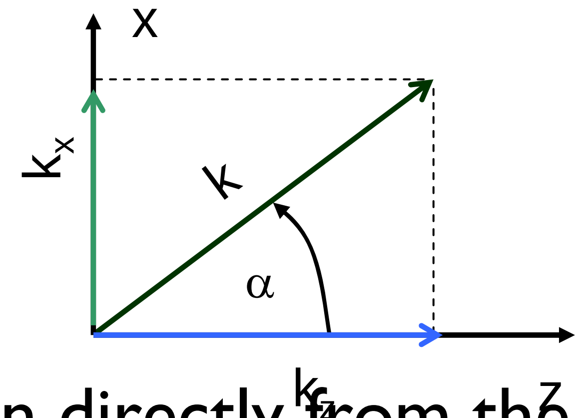
- Even though this equation is derived from small angels approximation, it is still valid for large angels.
- It is referred to as the *grating equation*.

Using k vector

- The grating equation can be re-written, by multiplying each side by 2π/λ, as

$$\frac{2\pi}{\lambda}\sin\alpha_m = \frac{2\pi}{\lambda}\sin\alpha_i + m\frac{2\pi}{a}$$

$$k_{x,m} = k_{x,i} + mK, \text{ where } K = \frac{2\pi}{a}$$



- This equation can be driven directly from the continuity of the tangential component, k_x, of the k vector.
- K is called the grating vector

Grating equation with k vector

$$k_{x,m} = k_{x,i} + mK$$

- The magnitude of km is constant

$$k_{z,m} = \sqrt{k_o^2 - k_{x,m}{}^2}$$

$$k_o = \frac{2\pi n}{\lambda}$$