Fundamentals of Optics

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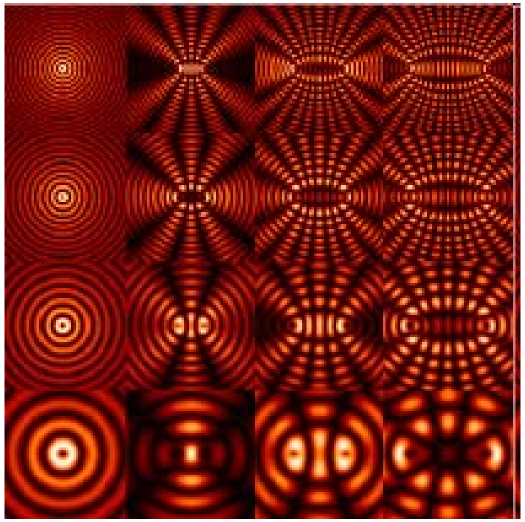
INTERFERENCE

The concept

- interference is the addition (superposition) of two or more waves that result in a new wave pattern.



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- interference usually refers to the interaction of waves which are correlated or ***coherent*** with each other

Addition (superposition)

- Simply interference is a superposition of two or more waves maintaining their relative phase information.

Field

$$g(x) = E_1(x) + E_2(x)$$

$$E_1(x) = |E_1(x)|e^{i\phi_1(x)}$$

$$E_2(x) = |E_2(x)|e^{i\phi_2(x)}$$

$$g(x) = |g(x)|e^{i\phi(x)}$$

Intensity

$$I(x) = |g(x)|^2$$

$$I(x) = |E_1(x) + E_2(x)|^2$$

$$I(x) = |E_1(x)|^2 + |E_2(x)|^2 +$$

$$2|E_1(x)||E_2(x)|\cos\left(\phi_1(x) - \phi_2(x)\right)$$

Intensity

- Remember eye sees only intensity.
- Intensity is generally defined as

$$I(x) = |g(x)|^2 = g(x)g(x)^*$$

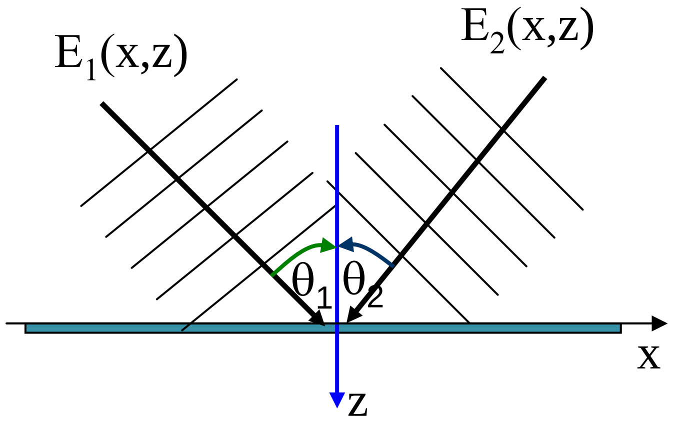
- When two waves interfere, the intensity is

$$I(x) = (E_1(x) + E_2(x))(E_1(x) + E_2(x))^*$$
$$= E_1(x)E_1(x)^* + E_2(x)E_2(x)^* + E_1(x)E_2(x)^* + E_2(x)E_1(x)^*$$
$$= |E_1(x)|^2 + |E_2(x)|^2 + |E_1(x)||E_2(x)|\left(e^{i(\phi_1-\phi_2)} + e^{-i(\phi_1-\phi_2)}\right)$$
$$= |E_1(x)|^2 + |E_2(x)|^2 + 2|E_1(x)||E_2(x)|\cos(\phi_1(x) - \phi_2(x))$$

Two plane waves interference

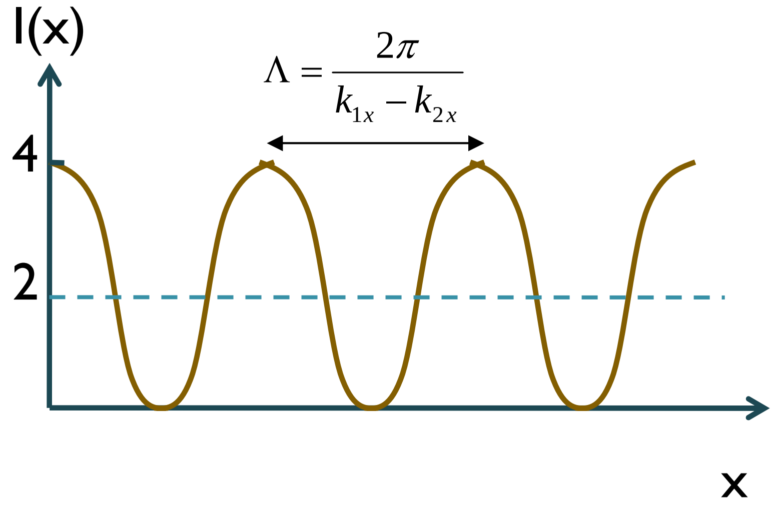
- The fields are

$$E_1(x,z) = 1 \cdot e^{i(k_{1x}x + k_{1z}z)}$$
$$E_2(x,z) = 1 \cdot e^{i(k_{2x}x + k_{2z}z)}$$



- The intensity at the plane z=0 is

$$I(x) = |E_1(x)|^2 + |E_2(x)|^2 + 2|E_1(x)||E_2(x)|\cos(\Delta\phi(x))$$
$$= 1 + 1 + 2 \cdot \cos((k_{1x} - k_{2x})x)$$
$$= 2 + 2 \cdot \cos((k_{1x} - k_{2x})x)$$

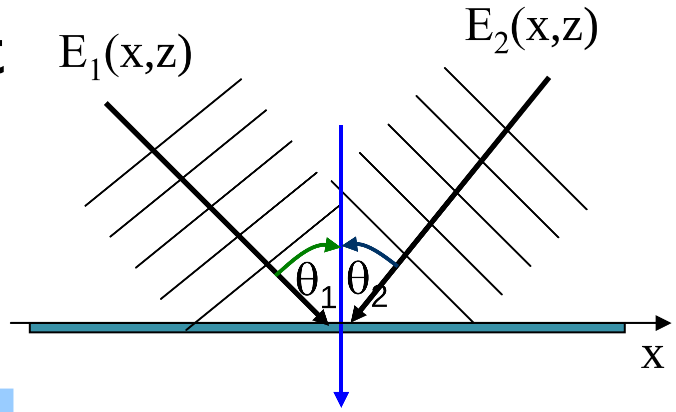


Interference example

- If two beams are incident at two different angles

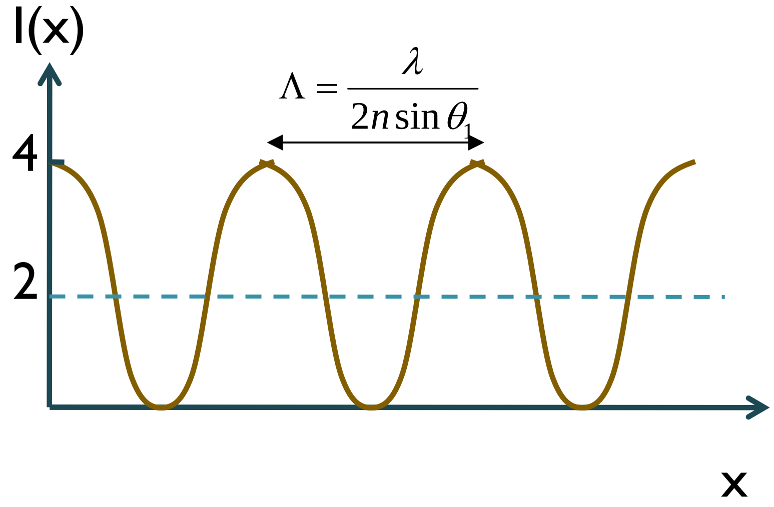
$$k_{1x} = k_o n \sin\theta_1$$
$$k_{2x} = -k_o n \sin\theta_2$$

$$I(x) = 2 + 2\cos\left(\frac{2\pi n}{\lambda}(\sin\theta_1 + \sin\theta_2)\right)$$



- If $\theta_1 = \theta_2$

$$I(x) = 2 + 2\cos\left(\frac{4\pi n}{\lambda}\sin\theta_1\right)$$



Exercise I

- Write the interference between a spherical wave and a plane wave.

- Write the interference pattern between two spherical waves coming from two point sources located in the same plane (at a distance z from the observation) and spaced by a distance d.

Interference and coherence

- Interference between two beams gives rise to a cosine term which presents the phase relation between the two beams in the intensity.
- The cosine term represent what is called fringes pattern in the intensity.
- Interference requires high level of phase correlation between the two beams. That is referred to as *coherence.*
- If phase is random, the net addition of the two beams over time would result into intensity addition only.

The necessity of coherence

- When the phase is 100% correlated between the beams, the interference intensity is

$$I(x) = |E_1(x)|^2 + |E_2(x)|^2 + 2|E_1(x)||E_2(x)|\cos(\Delta\phi(x))$$

- When the phase is not correlated between the beams (random) the interference intensity is

$$I(x) = |E_1(x)|^2 + |E_2(x)|^2$$

Coherence

- Hence, the general intensity equation is

$$I(x) = |E_1(x)|^2 + |E_2(x)|^2 + 2\gamma |E_1(x)||E_2(x)| \cos(\Delta\phi(x))$$

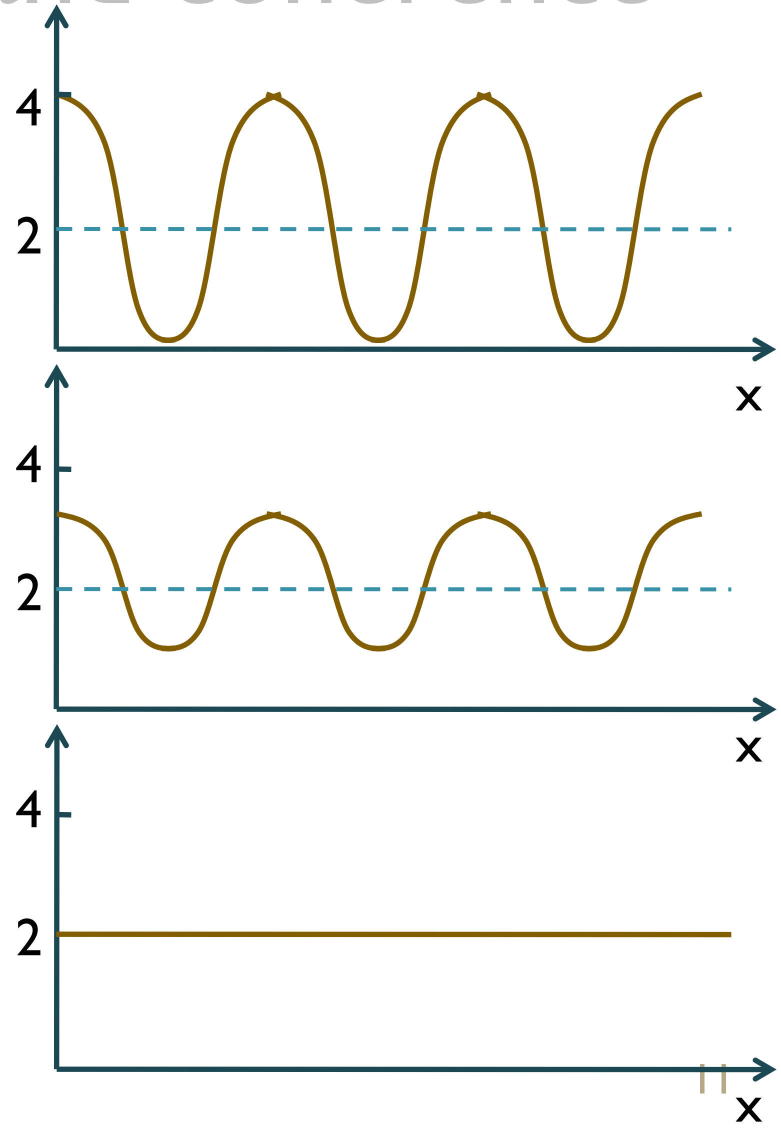
- γ is the coherence coefficient.
- $\gamma = 1$; the beams are totally coherent and the best fringes visibility is achieved.
- $\gamma = 0$; the beams are totally incoherent and no fringes are observed.
- $0 < \gamma < 1$; the beams are partially coherent and intermediate fringes visibility is achieved.

Fringes visibility and coherence

- Totally coherent
- $\gamma=1$

- Partially coherent
- $0<\gamma<1$

- Incoherent
- $\gamma=0$



Types of light coherence

- Coherence can be *temporal* or *spatial*.

- **Temporal coherence:** the phase between any two instances of time of the same beam are correlated.

- **Spatial coherence:** the phase between any two points across the light beam is correlated.

Laser coherence

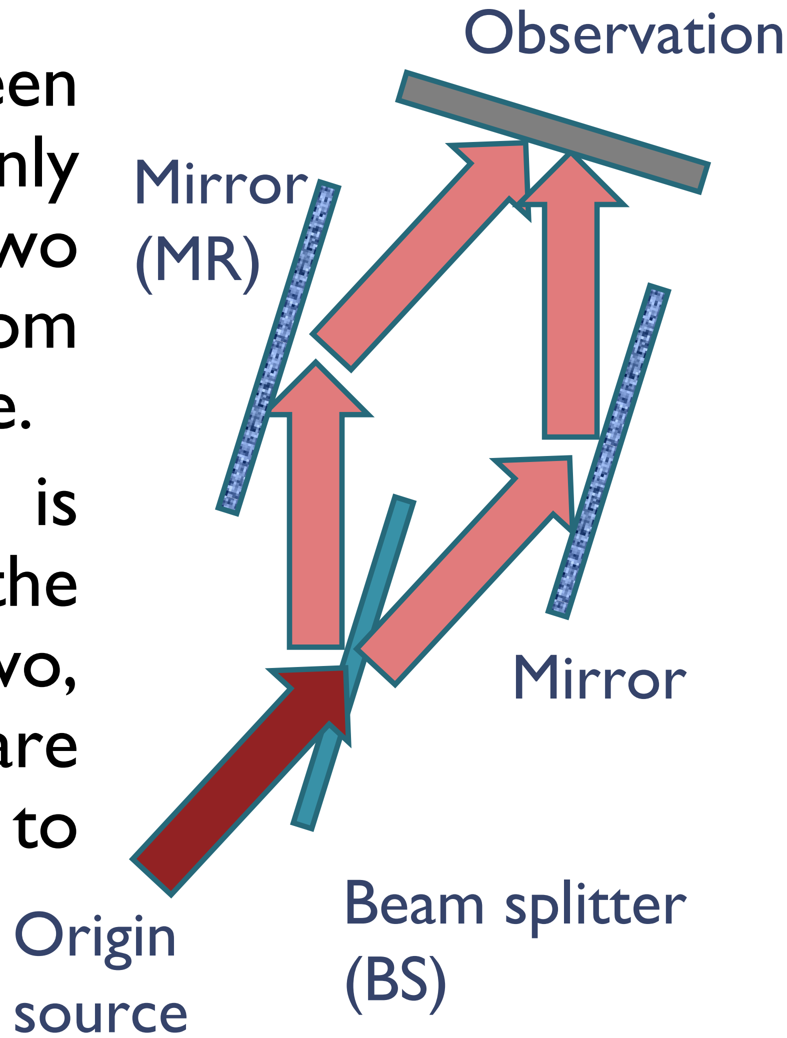
- Laser beam is assumed to be by default *totally spatially coherent.*
- Spatial coherence is reduced is the laser beam passes through a scattering medium.

- Laser is *not completely temporal coherent.*
- Temporal coherence is reduced with the spectral band width of the light.

- Two beams from **different sources** are **temporally incoherent**.

Interference devices

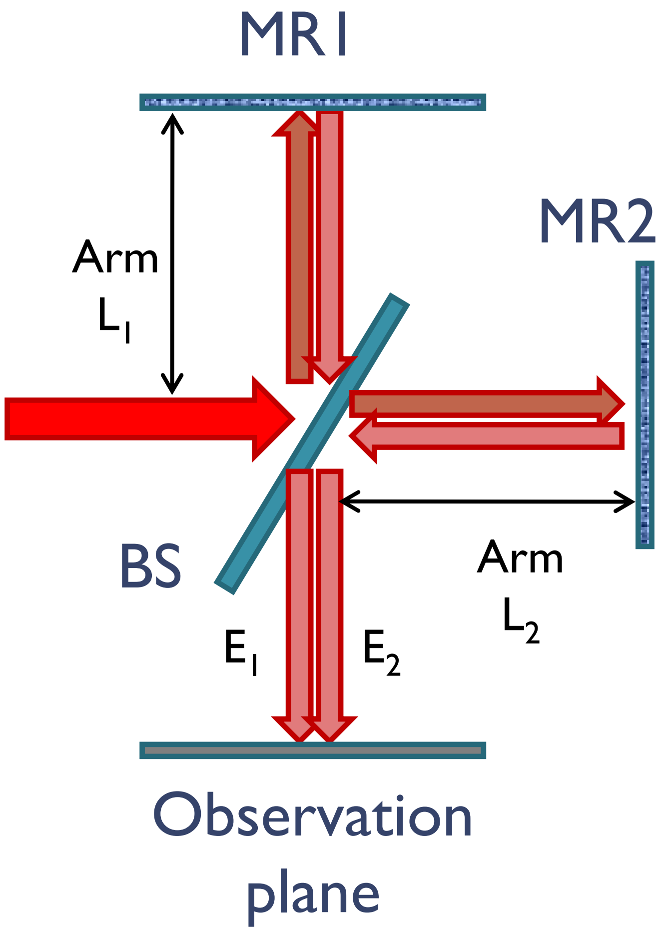
- Interference between two beams is only obtained if the two beams originate from the same laser source.
- A beam splitter is needed to split the input beam into two, where they are reflected by mirrors to interfere.



Interferometer

- Interferometer is a device that uses the interference between two (or more) beams for specific applications.

- One of the most famous interferometers is Michelson interferometer.



Schematic diagram of Michelson interferometer

Michelson interferometer

- The two fields at the observation plane are

$$E_1(x) = a_1 \cdot \exp\left[ik_o \cdot (2L_1 + L_o)\right]$$
$$E_2(x) = a_2 \cdot \exp\left[ik_o \cdot (2L_2 + L_o)\right]$$

- The intensity pattern at the observation plane is

$$I(x) = a_1^{\,2} + a_2^{\,2} + 2\gamma a_1 a_2 \cos\left(2k_o \cdot \left(L_2 - L_1\right)\right)$$

- The intensity pattern changes with the changes between the two arms (L_1 and L_2)

Michelson interferometer device

- If one arm (say L_1) is kept fixed.
- Any changes in the phase of the light passing through the arm L_2 causes a change in the intensity pattern.
- This setup can be used to sense the changes in the optical path in L2.
- That could be:
  - Vibration
  - Temperature change,
  - presence of different material and so on.

Sensing mechanism

- Vibration directly changes the value of L_2 by a small change ΔL.
- Change of temperature or presence of different material changes the refractive index of arm L_2.

- For temperature change: $\Delta L = (\Delta n - 1)L_2$
- For the presence of a different material of thickness t : $\Delta L = (\Delta n - 1)t$

- The effective arm length is : $L_2^{/} = L_2 + \Delta L$